2013 Q3

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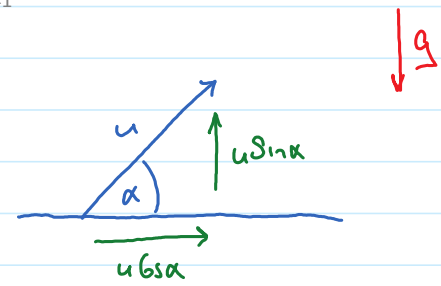
13:41



g ↓

$g_x = 0$

$g_y = g$

$V_x = u\cos\alpha - gt$ $\qquad$ $V_y = u\sin\alpha - gt$

$S_x = u\cos\alpha . t - \frac{1}{2}gt^2$ $\qquad$ $S_y = u\sin\alpha . t - \frac{1}{2}gt^2$

$S_y = 0$

$u\sin\alpha t - \frac{1}{2}gt^2 = 0$

$t = 0$ $\qquad$ $t = \frac{2u.\sin\alpha}{g}$

5M

$S_x \rightarrow$

$u.\cos\alpha\, t = R$

$u.\cos\alpha\left(\frac{2u\sin\alpha}{g}\right) = R$

$\frac{2u^2\sin\alpha.\cos\alpha}{g} = R$

5M

ii)

$V_y = 0 \Rightarrow u\sin\alpha - gt = 0$

$t = \frac{u.\sin\alpha}{g}$

5M

$S_y = \frac{R}{4\sqrt{3}}$

$S_y = u\sin\alpha\, t - \frac{1}{2}gt^2$

$\frac{R}{4\sqrt{3}} = u\sin\alpha\left(\frac{u\sin\alpha}{g}\right) - \frac{1}{2}g\left(\frac{u^2.\sin^2\alpha}{g^2}\right)$

5M

$= \frac{u^2.\sin^2\alpha}{g} - \frac{1}{2}\frac{u^2.\sin^2\alpha}{g} = \frac{u^2\sin^2\alpha}{g}\left(1 - \frac{1}{2}\right)$

$$= \frac{u^2 \sin^2\alpha}{g} - \frac{1}{2}\frac{u^2 \sin^2\alpha}{g} = \frac{u^2 \sin^2\alpha}{g}\left(1 - \frac{1}{2}\right)$$

$$\frac{R}{4\sqrt{3}} = \frac{1}{2}\left(\frac{u^2 \sin^2\alpha}{g}\right)$$

$$\frac{2u^2 \sin\alpha \cos\alpha}{4\sqrt{3}\, g} = \frac{u^2 \sin^2\alpha}{2g}$$

$$\frac{4\cos\alpha}{4\sqrt{3}} = \sin\alpha$$

$$\frac{1}{\sqrt{3}} = \frac{\sin\alpha}{\cos\alpha} \quad \rightarrow \quad \tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$\frac{1}{\sqrt{3}} = \tan\alpha$$

$$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\alpha = 30^\circ$$ Sm (25)

b) Don't sub in $\tan^{-1}\frac{1}{2}$ until the end.

$u_x = u\cos\theta \qquad u_y = u\sin\theta$

$a_x = -g\sin\alpha \qquad a_y = -g\cos\alpha$

$S_x = u_x t - \frac{1}{2}a_x t^2 = R \qquad S_y = u_y t - \frac{1}{2}a_y t^2 = 0$

$S_y = 0 \Rightarrow u\sin\theta . t - \frac{1}{2} g\cos\alpha\, t^2 = 0$ Sm

$t = 0 \quad \swarrow \qquad \searrow \quad t = \frac{2u\sin\theta}{g\cos\alpha}$ Sm

$$\Rightarrow \quad R = u\cos\theta\left(\frac{2u\sin\theta}{g\cos\alpha}\right) - \frac{1}{2} g \sin\alpha\left(\frac{4u^2\sin^2\theta}{g^2\cos^2\alpha}\right)$$

$$= \frac{2u^2 \sin\theta . \cos\theta}{g\cos\alpha} - \frac{2u^2 \sin^2\theta . \sin\alpha}{g\cos^2\alpha}$$

Common Denom. $= g\cos^2\alpha$

$$= \frac{2u^2 \sin\theta \cos\theta \cos\alpha - 2u^2 \sin^2\theta . \sin\alpha}{g\cos^2\alpha}$$

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$$= \frac{2u^2 \sin\theta (\cos\theta . \cos\alpha - \sin\theta . \sin\alpha)}{g . \cos^2\alpha}$$

$\rightarrow \cos A+B = \cos A\cos B - \sin A\sin B$

$$= \frac{2u^2 \sin\theta . \cos(\theta+\alpha)}{g . \cos^2\alpha}$$

$\rightarrow 2\sin A\cos B = \sin(A+B) + \sin(A-B)$

↳ where $A = \theta$

$B = \theta + \alpha$

$$= \frac{u^2 [\sin(\theta+\theta+\alpha) + \sin(\theta-\theta-\alpha)]}{g . \cos^2\alpha}$$

$$= \frac{u^2}{g\cos^2\alpha} (\sin(2\theta+\alpha) + \sin(\alpha)]$$

$$= \frac{u^2}{g\cos^2\alpha} (\sin(2\theta+\alpha) - \sin\alpha)$$

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Max Value of Sin function involving $\theta = 1$

θ is angle of projection.

$\sin(2\theta+\alpha) = 1$

$2\theta + \alpha = 90$

$\theta = \frac{90-\alpha}{2}$

$\theta = 45 - \frac{1}{2}\alpha$ ..... $\tan^{-1}\frac{1}{2} = \alpha$

$\theta = 45 - \frac{1}{2}\alpha$ ..... $\tan^{-1}\frac{1}{2} = \alpha$

$\approx 26.6°$

$= 45 - \frac{1}{2}(26.6)$

$\theta = 31.7°$

SM

(25)