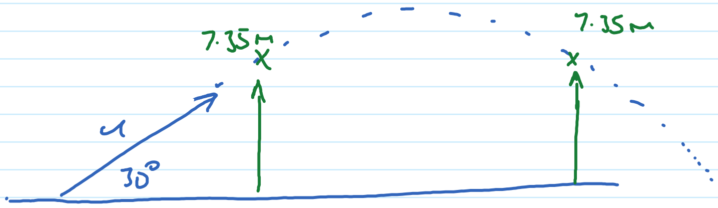
2014 Q3

27 April 2015
12:58

Q3 a)



$u_x = u\cos 30$ $\quad$ $u_y = u\sin 30$

$V_x = u\cos 30$ $\quad$ $V_y = u\sin 30 - gt$

$S_x = u\cos 30\, t$ $\quad$ $S_y = u\sin 30\, t - \frac{1}{2}gt^2$ $\quad$ 5m

When $S_y = 7.35$m

$$\Rightarrow 7.35 = \frac{u}{2}t - \frac{1}{2}gt^2$$ 5m

$$14.7 = ut - gt^2$$

$$gt^2 - ut + 14.7 = 0$$ 5m

$a = g$ $\quad$ -b form

$b = -u$

$c = 14.7$

$$t_1 = \frac{u - \sqrt{u^2 - 58.8g}}{2g} \qquad t_2 = \frac{u + \sqrt{u^2 - 58.8g}}{2g}$$

$$t_2 - t_1 = 1.5$$

$$\frac{u + \sqrt{} - (u - \sqrt{})}{2g} = 1.5$$

$$\frac{2\sqrt{}}{2g} = 1.5$$

$$\sqrt{u^2 - 58.8g} = 1.5g$$

$$u^2 - 58.8g = 2.25g^2$$

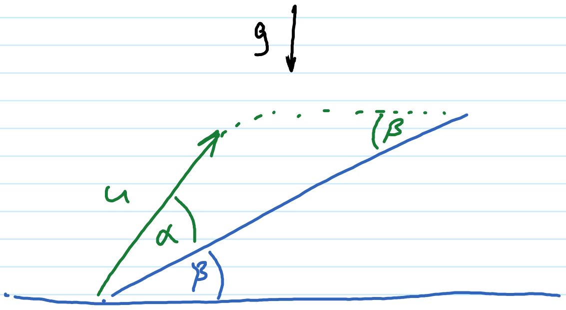
$$u = 792.33$$

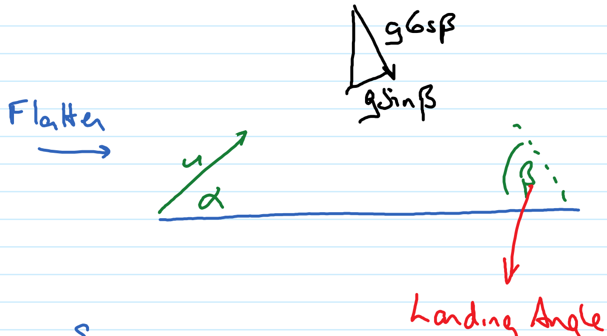
$$u = 28.15 \text{ m/s}$$

5m

20m

Q3 b)





$\tan \beta = \frac{-V_y}{V_x}$

$u_x = u \cos \alpha$ $\qquad$ $u_y = u \sin \alpha$

$V_x = u \cos \alpha - g \sin \beta \, t$ $\qquad$ $V_y = u \sin \alpha - g \cos \beta \, t$

$S_x = u \cos \alpha \, t - \frac{1}{2} g \sin \beta \, t^2$ $\qquad$ $S_y = u \sin \alpha \, t - \frac{1}{2} g \cos \beta \, t^2$

On impact: $S_y = 0$ $\qquad$ 5m

$$u \sin \alpha \, t - \frac{1}{2} g \cos \beta \, t^2 = 0$$

$t = 0$ or $t = \dfrac{2u \sin \alpha}{g \cos \beta}$ $\qquad$ 5m

Time of Impact.

On impact, landing angle is β.

$\therefore \tan \beta = \dfrac{-V_y}{V_x}$

Find V_y and V_x

If $t = \dfrac{2u\sin\alpha}{g\cos\beta}$ $\Rightarrow$ $V_y = u.\sin\alpha - g\cos\beta t$

$$= u.\sin\alpha - g\cos\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)$$

$$= u.\sin\alpha - 2u\sin\alpha$$

$$= -u.\sin\alpha \qquad 5M$$

$\Rightarrow$ $V_x = u.\cos\alpha - g\sin\beta t$

$$= u.\cos\alpha - g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)$$

$$= u.\cos\alpha - 2u\sin\alpha\tan\beta \qquad 5M$$

$$\tan\beta = \frac{-(-u\sin\alpha)}{u\cos\alpha - 2u\sin\alpha\tan\beta}$$

$$= \frac{\sin\alpha}{\cos\alpha - 2\sin\alpha\tan\beta}$$

$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

$\sin\alpha = \cos\alpha.\tan\alpha$

$$= \frac{\sin\alpha}{\cos\alpha - 2\cos\alpha.\tan\alpha.\tan\beta}$$

$$= \frac{\sin\alpha}{\cos\alpha(1 - 2\tan\alpha.\tan\beta)}$$

$$\tan\beta = \frac{\tan\alpha}{1 - 2\tan\alpha\tan\beta}$$

$\tan\beta - 2\tan\alpha\tan^2\beta = \tan\alpha$ ← We need $\tan\alpha = \dfrac{\tan\beta}{1+2\tan^2\beta}$

$\tan\beta = \tan\alpha + 2\tan\alpha\tan^2\beta$

$\tan\beta = \tan\alpha\,(1+2\tan^2\beta)$

$$\Rightarrow \frac{\tan\beta}{1+2\tan^2\beta} = \tan\alpha$$

QED

5m

ii)

$$1 + 2\tan^2\beta > 1$$

$$\therefore \tan\alpha = \frac{\tan\beta}{1+2\tan^2\beta} < \tan\beta$$

↑ $\tan\beta$ divided by number bigger than one means $\tan\alpha$ will always be bigger.