



# Coimisiún na Scrúduithe Stáit State Examinations Commission

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LEAVING CERTIFICATE EXAMINATION, 2007

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APPLIED MATHEMATICS – HIGHER LEVEL

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FRIDAY, 22 JUNE – AFTERNOON, 2.00 to 4.30

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Six questions to be answered. All questions carry equal marks.  
Mathematics Tables may be obtained from the Superintendent.  
Take the value of  $g$  to be  $9.8 \text{ m/s}^2$ .

**Marks may be lost if necessary work is not clearly shown.**

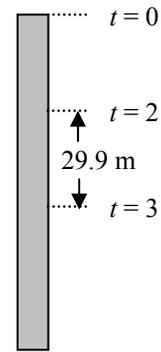
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1. (a) A particle is projected vertically downwards from the top of a tower with speed  $u$  m/s. It takes the particle 4 seconds to reach the bottom of the tower.

During the third second of its motion the particle travels 29.9 metres.

Find

- (i) the value of  $u$   
(ii) the height of the tower.



- (b) A train accelerates uniformly from rest to a speed  $v$  m/s.

It continues at this speed for a period of time and then decelerates uniformly to rest.

In travelling a total distance  $d$  metres the train accelerates through a distance  $pd$  metres and decelerates through a distance  $qd$  metres, where  $p < 1$  and  $q < 1$ .

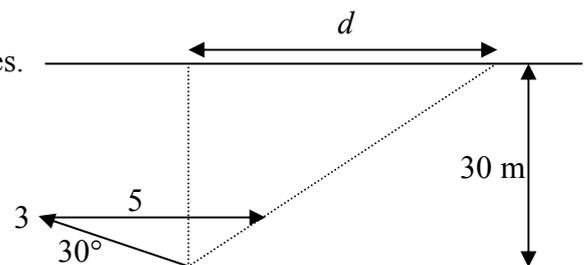
- (i) Draw a speed-time graph for the motion of the train.  
(ii) If the average speed of the train for the whole journey is  $\frac{v}{p+q+b}$ , find the value of  $b$ .

2. (a) Ship B is travelling west at 24 km/h. Ship A is travelling north at 32 km/h.

At a certain instant ship B is 8 km north-east of ship A.

- (i) Find the velocity of ship A relative to ship B.  
(ii) Calculate the length of time, to the nearest minute, for which the ships are less than or equal to 8 km apart.

- (b) A man can swim at 3 m/s in still water. He swims across a river of width 30 metres. He sets out at an angle of  $30^\circ$  to the bank. The river flows with a constant speed of 5 m/s parallel to the straight banks. In crossing the river he is carried downstream a distance  $d$  metres.



Find the value of  $d$  correct to two places of decimals.

3. (a) A particle is projected with a speed of  $7\sqrt{5}$  m/s at an angle  $\alpha$  to the horizontal.

Find the two values of  $\alpha$  that will give a range of 12.5 m.

- (b) A plane is inclined at an angle  $45^\circ$  to the horizontal. A particle is projected up the plane with initial speed  $u$  at an angle  $\theta$  to the **horizontal**. The plane of projection is vertical and contains the line of greatest slope.

The particle is moving horizontally when it strikes the inclined plane.

Show that  $\tan \theta = 2$ .

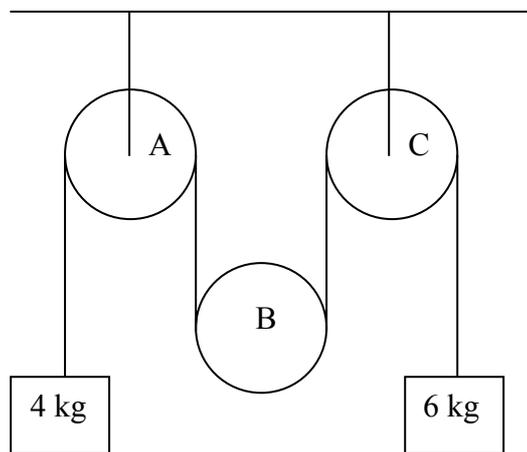
4. (a) A particle slides down a rough plane inclined at  $45^\circ$  to the horizontal. The coefficient of friction between the particle and the plane is  $\frac{3}{4}$ .

Find the time of descending a distance 4 metres from rest.

- (b) A light inextensible string passes over a small fixed pulley A, under a small moveable pulley B, of mass  $m$  kg, and then over a second small fixed pulley C.

A particle of mass 4 kg is attached to one end of the string and a particle of mass 6 kg is attached to the other end.

The system is released from rest.

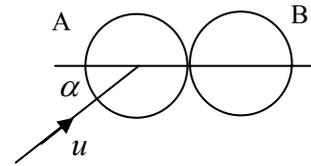


- (i) On separate diagrams show the forces acting on each particle and on the moveable pulley B.
- (ii) Find, in terms of  $m$ , the tension in the string.
- (iii) If  $m = 9.6$  kg prove that pulley B will remain at rest while the two particles are in motion.

5. (a) A smooth sphere P, of mass 2 kg, moving with speed 9 m/s collides directly with a smooth sphere Q, of mass 3 kg, moving in the same direction with speed 4 m/s.  
The coefficient of restitution between the spheres is  $e$ .

- (i) Find, in terms of  $e$ , the speed of each sphere after the collision.  
(ii) Show that the magnitude of the momentum transferred from one sphere to the other is  $6(1 + e)$ .

- (b) A smooth sphere A, of mass 4 kg, moving with speed  $u$ , collides with a stationary smooth sphere B of mass 8 kg.  
The direction of motion of A, before impact, makes an angle  $\alpha$  with the line of centres at impact.



The coefficient of restitution between the spheres is  $\frac{1}{2}$ .

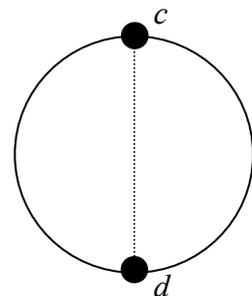
Find in terms of  $u$  and  $\alpha$

- (i) the speed of each sphere after the collision  
(ii) the angle through which the 4 kg sphere is deflected as a result of the collision  
(iii) the loss in kinetic energy due to the collision.

6. (a) A particle of mass  $m$  kg is suspended from a fixed point  $p$  by a light elastic string.  
The extension of the string is  $d$  when the particle is in equilibrium.  
The particle is then displaced vertically from the equilibrium position a distance not greater than  $d$  and is then released from rest.

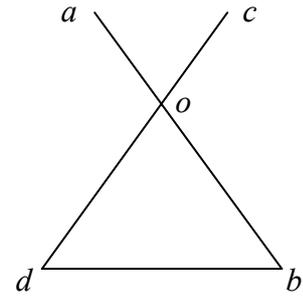
- (i) Show that the motion of the particle is simple harmonic.  
(ii) Find, in terms of  $d$ , the period of the motion.

- (b) A bead slides on a smooth fixed circular hoop, of radius  $r$ , in a vertical plane.  
The bead is projected with speed  $\sqrt{10gr}$  from the highest point  $c$ .  
It impinges upon and coalesces with another bead of equal mass at  $d$ .  
 $cd$  is the vertical diameter of the hoop.



Show that the combined mass will not reach the point  $c$  in the subsequent motion.

7. (a)  $ao$  and  $cod$  are two uniform rods, each of weight  $W$ , freely hinged at  $o$ .  
 $|ao| = |co| = 2\ell$  and  $|ob| = |od| = 5\ell$ .  
 The rods are in equilibrium in a vertical plane.

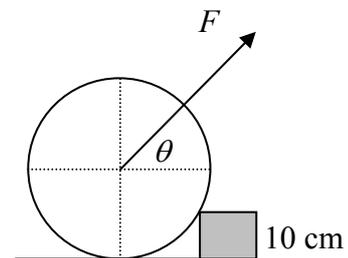


The ends  $b$  and  $d$  rest on a smooth horizontal plane and are connected by a light inextensible string of length  $5\ell$ .

Find the tension in the string.

- (b) A uniform disc of radius 25 cm and mass 100 kg rests in a vertical plane perpendicular to a kerb stone 10 cm high.

A force  $F$  is applied to the disc at an angle  $\theta$  to the horizontal, where  $\tan \theta = \frac{4}{3}$ .



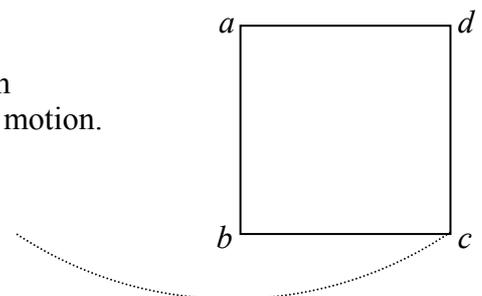
- (i) Draw a diagram showing all the forces acting on the disc.  
 (ii) Find the least value of  $F$  required to raise the disc over the kerb stone.

8. (a) Prove that the moment of inertia of a uniform square lamina, of mass  $m$  and side  $2r$ , about an axis through its centre parallel to one of the sides is  $\frac{1}{3}mr^2$ .

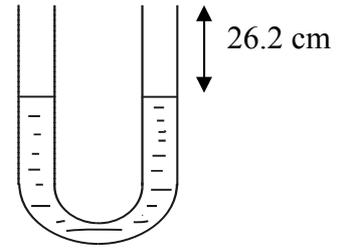
- (b) (i) A uniform square lamina  $abcd$  of side  $2r$  oscillates in its own plane about a horizontal axis through  $a$ , perpendicular to its plane.

If the period of small oscillations is  $2\pi\sqrt{\frac{8}{3g}}$ , find the value of  $r$ .

- (ii) If the lamina is released from rest when  $ab$  is vertical, find the maximum velocity of corner  $c$  in the subsequent motion.



9. (a) A U-tube whose limbs are vertical and of equal length has mercury poured in until the level is 26.2 cm from the top in each limb.



Water is then poured into one limb until that limb is full.

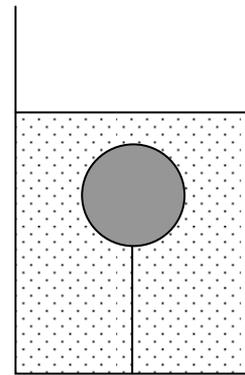
The relative density of mercury is 13.6.

Find the length of the column of water added to the limb.

- (b) A uniform solid sphere is held completely immersed in  $500 \text{ cm}^3$  of water by means of a string tied to a point on the base of the container.

The tension in the string is 0.0784 N.

When  $300 \text{ cm}^3$  of another liquid, of relative density 1.2 is added and thoroughly mixed with the water, the volume of the mixture is  $800 \text{ cm}^3$  and the tension in the string is 0.1078 N.



Find

- (i) the relative density of the mixture
- (ii) the mass of the sphere.

10. (a) Solve the differential equation

$$\frac{dy}{dx} = y^2 \sin x$$

given that  $y = 1$  when  $x = \frac{\pi}{2}$ .

- (b) The acceleration of a racing car at a speed of  $v$  m/s is

$$\left(1 - \frac{v^2}{3200}\right) \text{ m/s}^2$$

The car starts from rest.

Calculate correct to two decimal places

- (i) the speed of the car when it has travelled 1500 m from rest
- (ii) the maximum speed of the car.

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