**Question 10: Differential Equations**

***Please remember to photocopy 4 pages onto one sheet by going A3→A4 and using back to back on the photocopier***

|  |  |
| --- | --- |
|  | **Page** |
| Introduction | **2** |
| Revision of Logs | **3** |
| Common integration questions which involve logs | **4** |
| Exam questions: part ‘a’s | **5** |
| Exam questions: part ‘b’s | **9** |
| Motion expressed in terms of acceleration | **9** |
| Motion expressed in terms of forces | **12** |
| Power | **15** |
| Guide to answering exam questions: 2010 - 1996 | **16** |
|  |  |

**\*\*\*\*\*\*\*\*\*\*\* Marking Schemes / Solutions to be provided separately \*\*\*\*\*\*\*\*\*\*\*\*\***

**Introduction**

Up until now anytime we needed to connect v, u, a, s or t, we simply used our equations of motion.

These equations only work however if the acceleration is constant.

In this chapter we will see that the acceleration is not constant and therefore we need to resort to integration to help us.

Be comfortable using logs and integration (particularly integration by substitution) – see textbook.

For most questions you will use either **a= dv/dt** or **a = v.dv/ds**,

Occasionally you will need to link s and t directly so you must first use either a = v.dv/ds (to get an expression for v in terms of s) or a = dv/dt (to get an expression for v in terms of t) and then substitute ds/dt for v and continue.

Either all terms must represent an acceleration or a force, ***but you cannot mix them***.

If the left hand side is expressed as a force (e.g. mdv/dt) then all terms on the right hand side must be expressed as forces.

Similarly if the left hand side is given as an acceleration (e.g. dv/dt) then all terms on the right hand side must be expressed as accelerations.

Sometimes it’s not obvious whether a term in the question is an acceleration or a force: look at the units to help you.

If the resistance is expressed as kv2 N per unit mass then mathematically the resistive force = - mkv2 (it is a force).

**Should I use constants of integration or limits?**

It’s (slightly) interesting that the marking schemes usually use constants of integration for part (a) but tend to use limits of integration for part (b).

You can use either. Some students have a preference for one over the other.

**Revision of Logs  
Write in any unfamiliar relationships involving logsCommon integration questions which involve logs**   
**Write in any unfamiliar integrations** **Exam questions: part ‘a’s**

**The maths part**

* Generally when the question says ‘solve’, the convention seems to be solve for the variable on top (usually y).
* Begin by taking out common factors.
* Bring terms involving x to one side and terms involving y to the other.
* Ensure that dy and dx are on top on both sides.
* From 2012 onwards questions which require ‘Integration by Substitution’ will no longer be asked (it’s no longer on the Higher Maths course).
* Average speed =

**2012 (a)**

Newton’s law of cooling states that ‘the rate of cooling of a body is proportional to the difference between the temperature of a body and the temperature of its surroundings.’

If *θ* is the difference between the temperature of a body and the temperature of its surroundings then

A body cools from 80° C to 60° C in 10 minutes.

The temperature of the surroundings is maintained at 20° C.

Find

1. the value of *k*
2. the temperature of the body after a further 15 minutes.

**2011 (a)**

If and y = 1 when x = 1, find the value of y when x = 2.

**2010 (a)**

Solve the differential equation given that y = 0 when x = 0



**2009 (a)**

Solve the differential equation given that y = √3 when x =1.

**2008 (a)**

If  and y = 0 when x = 0, find the value of x when 

**2007 (a)**

Solve the differential equation  given that *y* = 1 when *x* = π/2

**2006 (a)**

Solve the differential equation  given that y = e when x = 0.

**2005 (a)**

Solve the differential equation – xy – y = 0 given that y = 1 when x = 1.

**2004 (a)**

Solve the differential equation  given that y = 5 when x = 1.

**2003 (a)**

Solve the differential equation given that y = 1 when x = √2.

**2002 (a)**

Solve the differential equation  given that y = ln 4 when x = 0.

**2001 (a)**

Find.

Hence, or otherwise, solve the differential equationgiven that y = 1 when x = 1.

**2000 (a)**

If  and y = 2 when x = e, find, correct to two places of decimals, the positive value of y when x = e2.

**1999 (a)**

Solve the differential equation given that *v* = 0 when *x* = 1.

**1998 (a)**

If t = v – vt and v = 3 when t = 5, find the value of v when t = 6.

**1997 (a)**

If  and y = 2 when x = 0, find, correct to two places of decimals, the value of y when x = 1.

**1996 (a)**

Solve the differential equation  if *y* = *e*2 when 

**1995 (a)**

Solve the differential equation  if *x* = 0 when *y* = 1.

**1994 (a)**

Solve the differential equation  if *y* = 1 when *x* = 0.

**1993 (a)**

If  and *y* = 2 when *x* = 1, find the value of *y* when *x* = 2.

**1992 (a)**

If  and if *y* = 1 when *x* =, find the value of *y* when *x* = .

**1991 (a)**

Solve the differential equation  if *y* = 1 when *x* = 1.

**1990 (a)**

Solve the differential equation  if *x* = 1 when *y* = 1.

#### 1989 (a)

Find the solution of the differential equation  if *y* = 1 when *x* = 1.

**1988 (a)**

Solve the differential equation  if *x* = 5 when *t* = 0.

**1987 (a)**

Solve the differential equation  if *x* = 2 when *y* = 3

**1986 (a)**

Solve the differential equation  if *y* = 3 when *x* = 1.

**1985 (a)**

Find the solution of the differential equation if y = 0 when x = 2.

**1984 (a)**

Find the general solution to = *g* – *kv* where *g* and *k* are constants. Show that Lim v 

**1983 (a)**

Find the solution of the differential equation sin*x* = *y*cos*x* when *y* = 2 at *x* = .

**1982 (a)**

1. Find the solution of the differential equation  when *y* = 2 at *x* = 1.
2. Find the solution of the differential equation when = 1 at *t* = 0 and *s* = 0 at *t* = 0.

**1981 (a)**

Solve the differential equation given that y = 2 when x = π/6

**1980 (a)**

Solve the differential equation  given that  and x =  when y = 1.

**1979 (a)**

Solve the differential equation  hence or otherwise solve 

where *y* = 0 when *x* = 1 and *y* = 3 when *x* = *e*

**1978 (a)**

Solve the following differential equations:

1.  if *y* = 0 when *x* = 1
2.  if = 1 and *x* = ½ when *y* = 1

**1977**

1. Solve the differential equation  if *y* =  when *x* = 
2. Solve the equation  if *y* = 2 and  when *x* = 0.

**1974 (full question)**

A particle of mass 0·1 kg falls vertically from rest under gravity in a medium which exerts a resisting force of magnitude 0·02*v* newtons when the speed of the particle is *v* m/s.

Show that *v* = 49 and find *v*.



lim

Find an expression for the distance travelled in time t seconds.

**1973 (full question)**

An engine pulls a train along a level track against a resistance which at any time is *K* times the momentum. The engine works at constant power 25*K*MUo2, where M is the total mass of the train and engine and K, Uo are constants.

Show that the equation of motion of the train is *v*= *K*(25Uo2 – *v*2), and find the time taken to increase speed from Uo to 4Uo.

**1972 (full question)**

A particle of mass M kg is projected vertically upwards from ground level with a speed of 70 m/s.

In addition to the weight of the particle, there is the resistance force of the air of magnitude 1/20M*v*2 newtons when the speed is *v* metres per second.

Show that the equation of motion during the upward journey is –20

Prove that the maximum height reached is (10 ln 26)m, and that the time taken to reach it is approximately 1.95s. (Note: ln26 = loge 26)

**1971 (full question)**

A particle of mass M is projected with speed *u* along a smooth horizontal table.

The air resistance to motion when the speed of the particle is *v*, is M*kv* where *k* is a constant.

By solving the equation of motion for the particle, show that *v* = *ue*-*kt* and prove that as time increases indefinitely, the distance travelled ultimately approaches *u*/*k*.

Find the time taken for the particle to travel half this distance.

**Part ‘b’s**

**Motion expressed in terms of acceleration**

**2010 (b)**

The acceleration of a cyclist freewheeling down a slight hill is 0.12 – 0.0006*v*2 m s-2 where the velocity *v* is in metres per second.

The cyclist starts from rest at the top of the hill.

Find

1. the speed of the cyclist after travelling 120 m down the hill
2. the time taken by the cyclist to travel the 120 m if his average speed is 2.65 m s-1.

**2007 (b)**

The acceleration of a racing car at a speed of *v* m/s is m/s2.

The car starts from rest.

Calculate correct to two decimal places

1. the speed of the car when it has travelled 1500 m from rest
2. the maximum speed of the car.

**2009 (b)**

A particle of mass m is projected vertically upwards with speed u. The air resistance is kv2 per unit mass when the speed is v.

The maximum height reached by the particle is ln 4/(2k).

1. Find the value of u in terms of k.
2. Find the value of k if the time to reach the greatest height is π/3 seconds.

**2006 (b)**

The acceleration of a particle moving horizontally in a straight line is 1/x3 away from a fixed point o, where x is its distance from o. The particle starts from rest at x = 1.

1. Calculate the velocity of the particle when x = 4/3
2. Calculate the time that it takes the particle to reach a point 2 metres from o.

**2002 (b)**

A particle starts from rest and moves in a horizontal line.

Its speed v at time t is given by the equation .

1. Find the time taken for the speed of the particle to increase from 25 m/s to 75 m/s.
2. How far does the particle travel in going from rest to a speed of 75 m/s?
3. Determine the limiting speed, v1 of the particle ( that is v → v1 as t → ∞ ).

**2000 (b)**

The deceleration of a particle moving in a straight line with speed v m/s has magnitude  m/s2.

The particle has an initial speed of 6 m/s.

1. Find the time t1 for the speed to decrease to 3 m/s.
2. Find the time t2 for the particle to come to rest.
3. Deduce that.

**1998 (b)**

A particle moves in a straight line. The initial speed is *u* and the retardation is *kv*3, where *v* is the speed at the time *t*. If s is the distance travelled in time *t*, prove

1. 
2. 

**1997 (b)**

A particle moves in a straight line and undergoes a retardation of 0.04*v*3 m/s2, where *v* is its speed.

1. If the initial speed of the particle is 25m/s, find its speed when it has travelled a distance of 49 m.
2. Find the time for the speed to reduce from 25m/s to 15m/s.

**1989 (b)**

A cyclist, free-wheeling on a straight level road, experiences a retardation which is proportional to the square of his speed.

His speed is reduced from 6 m/s to 3 m/s in a distance of 35 m.

Show that the average speed during this period is 6*ln*2.

**1986 (b)**

A particle moves in a straight line in a medium whose resistance is proportional to the cube of its speed.

No other force acts on the body.

The speed falls from 15 m/s to 7**.**5 m/s in a time of *t* seconds. Show that the distance travelled in this time is 10*t* m.

**1985 (b)**

A particle of mass m moves in a straight line.

The only force acting on it being a resistance mkv2, where v is its speed and k is a constant.

It is initially projected from the point o with speed u.

When the particle reaches a point p on the line its speed is u/3.

1. Show that the average speed between o and p is ½ ulog3.
2. Find the speed of the particle when it is at the midpoint of [op].

**1984 (b)**

A car, free-wheeling on a straight road, experiences a retardation which is proportional to the square of its speed.

Its speed is reduced from 20 m/s to 10 m/s in a distance of 100 m.

Calculate the time taken to travel the 100 m.

**1983 (b)**

A particle of mass 8 kg moves along a line (the x-axis) on a smooth horizontal plane under the action of a force in newtons of (40 – 3) where  is the unit vector along the axis and *x* is the displacement of the particle from a fixed point *o* of the axis.

If the particle starts from rest at *o*, find its speed when *x* = 100 and calculate when it next comes to instantaneous rest.



**1982 (b)**

1. Find the solution of the differential equation when = 1 at *t* = 0 and *s* = 0 at *t* = 0.
2. A particle moves in a straight line with acceleration equal to minus the square of its velocity.

If its initial velocity is 1 m/s, calculate the distance travelled one second later.

**1981 (b)**

1. Find the general solution to  where K is a constant.
2. A particle moves in a straight line so that at any instant its acceleration is, in magnitude, half its velocity. If its initial velocity is 3 m/s, find an expression for the distance it describes in the fifth second.

**1980 (b)**

A car starts from rest.

When it is at a distance *s* from its starting point, its speed is *v* and its acceleration is 5 – *v*2.

Show that *vdv* = (5 – *v*2)*ds* and find as accurately as the tables allow its speed when *s* = 1.5.

**1979 (b)**

A body is moving in a straight line subject to a deceleration which is equal to , where *v* is the velocity. The initial velocity is 5 m/s.

In how many seconds will the velocity of the body be 2 m/s and how far will it travel in that time?

**1978 (b)**

A particle of mass *m* is acted on by a force 2*m*/*x*5 directed away from a fixed point *O*, where *x* is the distance of the particle from *O*.

The particle starts from rest at a distance *d* from *O*.

Show that the velocity of the particle tends to a limit .

**Motion expressed in terms of forces**

**2012 (b)**

A particle of mass *m* is fired horizontally through a block of resistive gel.

The resistance to motion is *mkv2* N when *v* m s–1 is the speed.

The particle enters the gel at a speed of 1000 m s–1 and seconds later exits the gel at a speed of 10 m s–1.

1. Show that *k* =
2. Use the fact that = or otherwise to show that the length of the block of gel is ln100 m.

**2005 (b)**

A mass of 9 kg is suspended at the lower end of a light vertical rope.

Initially the mass is at rest. The mass is pulled up vertically with an initial pull on the rope of 137.2 N.

The pull diminishes uniformly at the rate of 1 N for each metre through which the mass is raised.

1. Show that the resultant upward force on the mass when it is x metres above its initial position is 49 − x .
2. Find the speed of the mass when it has been raised 15 metres.
3. Find the work done by the pull on the rope when the mass has been raised by 15 m.

**2004 (b)**

A particle is projected vertically upwards with an initial speed of 2g m/s in a medium in which there is a resistance kv2 N per unit mass where v is the speed of the particle and k is a constant, where k > 0.

1. Prove that the maximum height reached is 
2. If the speed of the particle is g m/s when it has reached half its maximum height, find the value of k.

**2001 (b)**

A car of mass m kg is travelling along a level road. The resistance to motion is mkv2 N, where v m/s is the speed. When the car is travelling at 14 m/s, the engine cuts out.

Ten seconds after the engine cuts out, the speed of the car is 7 m/s.

1. Show that k =.
2. The car travels a distance of s metres in the first T seconds after the engine cuts out.

Show that 

**1999 (b)**

The rocket engine of a 12 tonne missile produces a thrust of 180.1 kN. The missile is launched in a vertical direction. The air resistance is *v*2 N where *v* is the speed of the missile.

1. Find the speed of the missile after 30 seconds.
2. Find the percentage error in this speed if air resistance is ignored.

**1996 (b)**

A particle of mass *m* is projected vertically upwards with a velocity *v* of , the air resistance being *kv*2 per unit mass. Prove

1. the greatest height reached by the particle is 
2. the velocity of the particle when passing through the point of projection on the way down is 

**1995 (b)**

A particle of mass *m* falls from rest against air resistance of *mkv*, where *k* is constant and *v* is the speed. Prove that

1. the time taken to acquire a speed of  is 
2. the speed of the particle tends to a limit 

**1993 (b)**

A particle starts with a speed of 20 m/s and moves in a straight line.

The particle is subjected to a resistance which produces a retardation which is initially 8 m/s2 and which increases uniformly with the distance moved, having a value of 9 m/s2 when the particle has moved a distance 5 m.

If *v* m/s is the speed of the particle when it has moved a distance *x* m

1. prove that, while the particle is in motion 
2. calculate the distance moved by the particle in coming to rest.

**1992 (b)**

A particle experiences a retardation of k*v* m/s2 when its velocity is *v* m/s. Its velocity is reduced from its initial value of 210 m/s to 70 m/s in 0**.**5 s and it travels a distance *x* m in this time.

1. Find the value of k and deduce an expression for the velocity at any time *t*.
2. Calculate the value of *x*.

**1991 (b)**

A particle is projected in a straight line from a fixed point with velocity *u* at time *t* = 0. It is opposed by a resistance *kvn* per unit mass. If s is the displacement at time *t* prove that when *v* = 0 is *n* < 1.

**1990 (b)**

A particle of mass 8 kg starts from rest and is acted on by a force which increases uniformly in 10 s from zero to 16 N.

1. Prove that *t* seconds after the particle begins to move, its acceleration is  m/s2.
2. Prove that, when the particle has moved *x* m, its speed is *v* m/s, where 10*v*3 = 9*x*2.

**1988 (b)**

A particle of mass *m* is projected vertically upwards with speed 120 m/s in a medium where there is a resistance of 0**.**098v2 per unit mass of the particle when *v* is the speed.

Calculate the time taken to reach the highest point.

**1987 (b)**

The resistance to motion of a train of mass *m* is constant and equal to 60 N per tonne.

When moving with constant speed 16 m/s on a level line the train begins to ascend an incline of 1 in 98, i.e. sin-1(1/98).

1. Assuming that the engine continues to work at the same rate (ie power is constant) and that *v* m/s is the speed of the train up the incline *t* seconds after the train has begun to climb, show the equation of motion is 
2. Calculate the time which elapses before the velocity falls to 12 m/s.

(Tables P.29 may be needed)

**1976 (full question)**

An atomic nucleus of mass M is repelled from a fixed point *o* by a force M *k*2*x*-5, where *x* is the distance of the nucleus from *o* and *k* is a constant.

It is projected directly towards o with speed  from a point *a* where |*oa*| = *d*.

Find the speed of the nucleus when if reaches the midpoint of *oa* and find how near it gets to *o*.

**1975**

The force of attraction of the earth on a particle of mass M distance *x* from the centre of the earth is , where *a* is the radius of the earth .

Write down the equation of motion for a particle moving under this force alone and calculate the speed of the particle at distance *x* if it was projected vertically upwards from the earth’s surface with speed .

Prove that the time taken to reach a height 3*a* above the earth’s surface is .

**1974 (full question)**

A particle of mass 0·1 kg falls vertically from rest under gravity in a medium which exerts a resisting force of magnitude 0·02*v* newtons when the speed of the particle is *v* m/s.

Show that *v* = 49 and find *v*.



lim

Find an expression for the distance travelled in time t seconds.

**Power**

Work = Force × displacement

W = F.s

P = F.v (Divide both sides by time)

**2008 (b)**

A train of mass 200 tonnes moves along a straight level track against a resistance of 400*v*2, where *v* m/s is the speed of the train.

The engine exerts a constant power of *P* kW.

The acceleration of the train is .

1. Find the value of *P*.
2. The train travels a distance 69.07 m while its speed increases from 10 m/s to *v*1 m/s. Find the value of *v*1.

**2003 (b)**

A car of mass 490 kg moves along a straight level horizontal road against a resistance of 70v N, where v m/s is the speed of the car. The engine exerts a constant power of 63 kW.

1. Show that the equation of motion is .
2. Calculate, correct to two decimal places, the time it takes the car to increase its speed from 10 m/s to 20 m/s.

**1994 (b)**

A car of mass 1000 kg moves with velocity *v* m/s along a horizontal road against a constant resistance of 1500 N.

The engine is working at a constant rate of 75 kW.

1. Show that the acceleration of the car is  m/s2.
2. Calculate, correct to two decimal places, the time taken by the car to increase its speed from 0 m/s to 25 m/s.

**Guide to answering the exam questions**

**2011 (a)**

1. Straightforward once you figured out how to rearrange the terms to solve the integration.   
   Answer: distance = 44.63 m
2. Straightforward. Answer: average speed = 19.24 m s-1

**2011 (b)**

**2010 (a)**

Straightforward. Answer:

**2010 (b)**

1. Straightforward to set up but the differentiation is difficult.

Answer: v = 5.18 m s-1

1. Straightforward. Answer: t = 45.3 s

**2009 (a)**

Straightforward. Answer: y = √(4x2 – 1)

**2009 (b)**

1. Care needs to be taken with the integration and algebra, but it’s nothing that hasn’t been seen before and is fairly straightforward. Answer: u = √(3*g*/k)
2. This time you are given a time so you must go back to the start and use dv/dt. The integration this time is particularly nasty. Solve to get k = 1/*g*.

**2008 (a)**

Easy peasy. Ans: x = 1.

**2008 (b)**

1. You need to know that Force = Power/velocity.

Ans: P = 3200 Watts

1. Straightforward.

Ans: v = 15 m s-1.

**2007 (a)**

Easy peasy. Ans: y = 1/(1+ Cos x)

**2007 (b)**

1. Need to use integration by substitution.

Ans: v = 44.12 m/s

1. Easy peasy if you remember that at maximum speed the acceleration will be zero, so use this.

Ans: v = 56.57 m/s.

**2006 (a)**

Need to use long division.

Ans: y = ex-ln(1+x)+1

**2006 (b)**

1. Easy peasy. Ans: v = 0.66 m/s.
2. There is no expression that links time and distance so we need to first get an expression that links velocity and distance (which we already have from part (i)), from that we get v = √((x2 – 1)/x2). Now substitute dx/dt for v and finish.

Ans: t = 1.73 seconds.

**2005 (a)**

Easy peasy. Ans: y = ex + ln x -1

**2005 (b)**

1. Easy if you’ve come across it before, not so easy if you haven’t. For every metre that the mass is raised the force will decrease by 1 Newton, so after x metres it will have decreased by x Newtons. So begin with Force = 137.2 – x – 9g and simplify to get the desired answer.
2. Straightforward. v = 11.76 m/s.
3. It’s not a concept we come across regularly (but we should – it also came up under Rigid Body Motion 2002 (b)); the energy something has is equivalent to the work done on it. So in this case to calculate the work done we need to calculate the energy which the mass has. The marking scheme uses the Kinetic Energy gained, but I don’t understand why you don’t also add the Potential Energy gained. On the likelihood that I am missing something we will go with their answer. If you can explain it please get back to me.

Ans: Work done = 622.5 J

**2004 (a)**

Straightforward. Ans: y = e-(1/x)+1

**2004 (b)**

1. Straightforward, but involves quite a bit of algebra to get the desired expression.
2. Start off the same as for the previous part, but the limits for x are now h/2 and h, and the limits for v are 2g and g. Even more algebra this time.

Ans: k = 2/g.

**2003 (a)**

Straightforward. Ans: y = (2x2 – 3)1/4

**2003 (b)**

1. Remember that Force = Power/velocity and the rest should follow.
2. Straight-forward. Ans: t = 1.65 seconds.

**2002 (a)**

Straight-forward. Ans: y = ln(ex + 3)

**2002 (b)**

1. Straight-forward. Ans: t = 1.1 seconds.
2. Straight-forward. Ans: x = 63.63 m.
3. Acceleration = 0, so v = 100 m.

**2001 (a)**

Apparently this question shouldn’t have got asked because the maths is not on the leaving cert honours syllabus.

**2001 (b)**

1. Straightforward.
2. You can’t link s and t directly, so you must first use v dv/ds as before to get v = 14 e-ks and then substitute ds/dt for v and continue.

Ans: s = 140 ln(1+T/10)

**2000 (a)**

Straightforward. y = 2.32

**2000 (b)**

1. Straightforward. Answer: t1 = 1.5(1/√e – 1/e)
2. Straightforward. Answer: t2 = 1.5(1 – 1/e)
3. Straightforward, albeit with some tricky algebra.

**1999 (a)**

1. Straightforward once you can deal with the integration; 1/(v2 + 1) becomes tan-1 v.
2. Answer: v = tan(ln x/7)

**1999 (b)**

Some quite tricky parts here.

1. Set it up as normal (draw a diagram to help you identify all forces) then use F(net) = ma.

Note that 180.1 kN = 180100 Newtons and 12 tonne = 12000 g, so you get 180100 – 12000 g – v2 = 12000 dv/dt

Now use the fact that (180100 – 12000 g) = 62500 which in turn (and here’s the tricky bit) = 2502.

Next use log tables to help you integrate, but if you’ve got this far you should be well able to bring it on home.

Answer: v = 138.64 m/s.

1. If air resistance is omitted then you can simply use equations of motion rather than differential equations (after all, air resistance is the only reason that we need this chapter in the first place). This gives a new value for v of 156.26 m/s.

Now you must also know that the formula for percentage error is “error/correct value, all multiplied by 100”.

Answer: percentage error = 12.7 %

**1998 (a)**

Straightforward

**1998 (b)**

1. Straightforward – nice question actually
2. Straightforward

**1997 (a)**

Straightforward

Answer: y = 0.82

**1997 (b)**

1. Straightforward

Answer: v = 0.5 m/s

1. Straightforward

Answer: t = 0.035 seconds

**1996 (a)**

Straightforward

Answer: y = e4 sin x

**1996 (b)**

1. Tricky integration to begin with, then tricky algebra with logs.
2. The question is basically asking you to find v when x = ln3/2k. Remember that for the equation at the beginning mg will be positive while the air resistance will be negative.