Chapter 8 Exercise 8A

Q. 1. (a) $R = 4\vec{j} + 5\vec{j} + \vec{j} = 10\vec{j}$ N

$4(0) + 5(50) + 1(100) = 10(x)$

$\Rightarrow x = 35$ cm

Answer: 35 cm from P

(b) $R = 2\vec{j} + \vec{j} + 2\vec{j} = 5\vec{j}$

$2(0) + 1(2) + 2(5) = 5x$

$\Rightarrow x = 2.4$ m

Answer: 2.4 m from P

(c) $R = \vec{j} - 7\vec{j} + \vec{j} = -5\vec{j}$

$1(0) - 7(1) + 1(2) = -5(x)$

$\Rightarrow x = 1$

Answer: 1 m from P

(d) $R = 3\vec{j} - 9\vec{j} + 3\vec{j} = -3\vec{j}$

$3(1) - 9(3) + 3(5) = -3x$

$\Rightarrow x = 3$

Answer: 3 m from P

(e) $R = \vec{j} - 2\vec{j} - 3\vec{j} + \vec{j} = -3\vec{j}$

$1(0) - 2(2) - 3(5) + 1(6) = -3(x)$

$\Rightarrow x = 4\frac{1}{3}$

Answer: $4\frac{1}{3}$ m from P

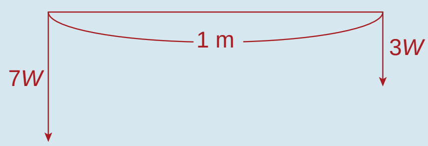
Q. 2. $R = 2W + W + 3W = 6W$

$2W(0) + W(3) + 3W(6) = 6W(x)$

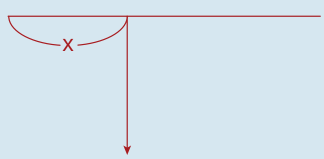
$\Rightarrow x = 3\frac{1}{2}$

Answer: $3\frac{1}{2}$ M and $2\frac{1}{2}$ M from the ends.

Q. 3. **Forces:**



Resultant:



$R = 7W + 3W = 10W$

$7W(0) + 3W(1) = 10W(x)$

$\Rightarrow x = \frac{3}{10}$ m

$= 30$ cm $=$ Answer

Q. 4. Let the length of the plank be 1.
Let d = the distance from the resultant's line of action from the left-hand end.

$R = W + x$

$\therefore W(0) + x(1) = (W + x)d$

$\Rightarrow d = \frac{x}{W + x}$

The remainder is $1 - \frac{x}{W + x} = \frac{W}{W + x}$

The ratio of these parts $= \frac{x}{W + x} : \frac{W}{W + x}$

$= x : W$

Q. 5. (i) $R = 4W + W + KW = (5 + K)W$

Taking moments about p.

$4W(0) + W(1) + KW(2) = (5 + K)W\left(\frac{7}{8}\right)$

$\Rightarrow 1 + 2K = \frac{7(5 + K)}{8}$

$\Rightarrow 8 + 16K = 35 + 7K$

$\Rightarrow K = 3$

(ii) $4W(0) + W(1) + KW(2) = (5 + K)W\left(\frac{11}{10}\right)$

$\Rightarrow 1 + 2K = \frac{11(5 + K)}{10}$

$\Rightarrow 10 + 20K = 55 + 11K$

$\Rightarrow K = 5$

Exercise 8B

Q. 1. $\left.\begin{array}{l} 3N \text{ at } (2, 1) \\ 2N \text{ at } (4, 3) \\ 1N \text{ at } (10, 9) \end{array}\right\} = 6N \text{ at } (x, y)$

$3(2) + 2(4) + 1(10) = 6(x)$

$\Rightarrow x = 4$

$3(1) + 2(3) + 1(9) = 6(y)$

$\Rightarrow y = 3$

Answer: (4, 3)

Q. 2. $\left.\begin{array}{l} 1N \text{ at } (1, 1) \\ 2N \text{ at } (1, 7) \\ 3N \text{ at } (3, 1) \\ 4N \text{ at } (2, 3) \end{array}\right\} = 10N \text{ at } (x, y)$

$1(1) + 2(1) + 3(3) + 4(2) = 10(x)$

$\Rightarrow x = 2$

$1(1) + 2(7) + 3(1) + 4(3) = 10(y)$

$\Rightarrow y = 3$

Answer: (2, 3)

Q. 3. $3(4) + 2(9) = 5x$

$\Rightarrow x = 6$

$3(1) + 2(-9) = 5y$

$\Rightarrow y = 3$

Answer: (6, −3)

$3(4) + 2(9) + 1(x) = 6(6)$

$\Rightarrow x = 6$

$3(1) + 2(-9) + 1(y) = 6(-2)$

$\Rightarrow y = 3$

Answer: (6, 3)

Q. 4. $W(3) + 2W(12) = 3W(x)$

$\Rightarrow x = 9$

$W(1) + 2W(19) = 3W(y)$

$\Rightarrow y = 13$

Answer: $(9, 13) = g$

$|pg| = \sqrt{(9-3)^2 + (13-1)^2}$

$= \sqrt{180}$

$= 6\sqrt{5}$

$|gq| = \sqrt{(12-9)^2 + (19-13)^2}$

$= \sqrt{45} = 3\sqrt{5}$

$|pg| : |gq| = 6\sqrt{5} : 3\sqrt{5}$

$= 2 : 1$

Q. 5.

$$\left.\begin{array}{l} 2N \text{ at } (1, 2) \\ 3N \text{ at } (1, 7) \\ 5N \text{ at } (5, -1) \end{array}\right\} = 10N \text{ at } (x, y)$$

$2(1) + 3(1) + 5(5) = 10(x)$

$\Rightarrow x = 3$

$2(2) + 3(7) + 5(-1) = 10(y)$

$\Rightarrow y = 2$

Answer: $3\vec{i} + 2\vec{j}$

$$\left.\begin{array}{l} 2N \text{ at } (1, 2) \\ 3N \text{ at } (1, 7) \\ 5N \text{ at } (5, -1) \\ 2N \text{ at } (x, y) \end{array}\right\} = 12N \text{ at } (4, 1)$$

$2(1) + 3(1) + 5(5) + 2(x) = 12(4)$

$\Rightarrow x = 9$

$2(2) + 3(7) + 5(-1) + 2(y) = 12(1)$

$\Rightarrow y = -4$

Answer: (9, −4)

Q. 6. $1(4) + 2(1) + 3(k) = 6\left(2\frac{1}{2}\right)$

$k = 3$

$1(1) + 2h + 3(1) = 6(4)$

$h = 10$

Q. 7. Centroid of triangle

$pqr = \left(\frac{2+5+3}{3}, \frac{1+3-1}{3}\right)$

$= \left(\frac{10}{3}, 1\right)$

$$\left.\begin{array}{l} M \text{ at } (2, 1) \\ M \text{ at } (5, 3) \\ M \text{ at } (3, -1) \end{array}\right\} = 3M \text{ at } (x, y)$$

$M(2) + M(5) + M(3) = 3M(x)$

$\Rightarrow x = \frac{10}{3}$

$M(1) + M(3) + M(-1) = 3M(y)$

$\Rightarrow y = 1$

The centre of gravity is at $\left(\frac{10}{3}, 1\right)$ which is the centroid.

$M(2) + M(5) + M(3) + 2M(x) = 5M(2)$

$\Rightarrow x = 0$

$M(1) + M(3) + M(-1) + 2M(y) = 5M(1)$

$\Rightarrow y = 1$

Answer: (0, 1)

Q. 8. (i) $\left.\begin{array}{l} W \text{ at } (6, 5) \\ 2W \text{ at } (7, -1) \\ 3W \text{ at } (2, 11) \\ 4W \text{ at } (6, 1) \end{array}\right\} = 10W \text{ at } (x, y)$

$W(6) + 2W(7) + 3W(2) + 4W(6) = 10W(x)$ … divide by W

$\Rightarrow 10x = 50$

$\Rightarrow x = 5$

$W(5) + 2W(-1) + 3W(11) + 4W(1) = 10W(y)$ … divide by W

$\Rightarrow 10y = 40$

$\Rightarrow y = 4$

$\Rightarrow$ The centre of gravity is at (5, 4)

(ii) $\left.\begin{array}{l} W \text{ at } (6, 5) \\ 2W \text{ at } (7, -1) \\ 3W \text{ at } (2, 11) \\ kW \text{ at } (6, 1) \end{array}\right\} = (6 + k)W \text{ at } (11, y)$

$W(6) + 2W(7) + 3W(2) + kW(6) = (6 + k)W(11)$ … divide by W

$\Rightarrow 66 + 11k = 26 + 6k$

$\Rightarrow 5k = -40$

$\Rightarrow k = -8$

$W(5) + 2W(-1) + 3W(11) - 8W(1) = -2W(y)$ … divide by W

$\Rightarrow 2y = -28$

$\Rightarrow y = -14$

Q. 9. $\left.\begin{array}{l} 2N \text{ at } (x, 5) \\ 3N \text{ at } (11, y) \\ 5N \text{ at } (5, -6) \end{array}\right\} = 10N \text{ at } (6, -5)$

$2(x) + 3(11) + 5(5) = 10(6)$

$\Rightarrow 2x + 58 = 60$

$\Rightarrow x = 1$

$2(5) + 3(y) + 5(-6) = 10(-5)$

$\Rightarrow 3y - 20 = -50$

$\Rightarrow y = -10$

Q. 10. $\left.\begin{array}{l} 1N \text{ at } (7, 8) \\ 4N \text{ at } (1, y) \\ xN \text{ at } (5, 2) \\ 2N \text{ at } (6, -3) \end{array}\right\} = (7 + x)N \text{ at } (4, 2)$

$1(7) + 4(1) + x(5) + 2(6) = (7 + x)(4)$

$\Rightarrow 5x + 23 = 28 + 4x$

$\Rightarrow x = 5$

$1(8) + 4(y) + 5(2) + 2(-3) = 12(2)$

$\Rightarrow 4y + 12 = 24$

$\Rightarrow y = 3$

Q. 11. $\left.\begin{array}{l} 3W \text{ at } (30, 20) \\ 2W \text{ at } (20, 10) \\ 5W \text{ at } (50, 20) \end{array}\right\} = 10W \text{ at } (x, y)$

$3W(30) + 2W(20) + 5W(50) = 10W(x)$
… divide by $10W$

$\Rightarrow x = 38$

$3W(20) + 2W(10) + 5W(20) = 10W(y)$
… divide by $10W$

$\Rightarrow y = 18$

$\Rightarrow$ Centre of gravity is at (38, 18)

Q. 12. $\left.\begin{array}{l} 3N \text{ at } \left(-\frac{7}{3}, 2\right) \\ 4N \text{ at } (1, 7) \\ 5N \text{ at } (3, 7) \\ 6N \text{ at } (4, -4) \end{array}\right\} = 18N \text{ at } (x, y)$

$3\left(-\frac{7}{3}\right) + 4(1) + 5(3) + 6(4) = 18(x)$

$\Rightarrow 18x = 36$

$\Rightarrow x = 2$

$3(2) + 4(7) + 5(7) + 6(-4) = 18(y)$

$\Rightarrow 18y = 45$

$\Rightarrow y = 2.5$

$\Rightarrow$ Centre of gravity is at $(2, 2.5)$

Q. 13. $\left.\begin{array}{l} 5 \text{ at } (3, -1) \\ 8 \text{ at } (4, 2) \\ 3 \text{ at } (-1, 5) \\ 2 \text{ at } (2, -6) \end{array}\right\} = 18 \text{ at } (x, y)$

$5(3) + 8(4) + 3(-1) + 2(2) = 18(x)$

$\Rightarrow 18x = 48$

$\Rightarrow x = \frac{8}{3}$

$5(-1) + 8(2) + 3(5) + 2(-6) = 18(y)$

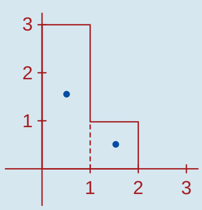
$\Rightarrow 18y = 14$

$\Rightarrow y = \frac{7}{9}$

$\Rightarrow$ Centre of gravity is at $\left(\frac{8}{3}, \frac{7}{9}\right)$

Exercise 8C

Q. 1. (i)



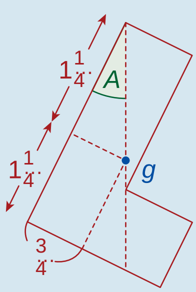
$\left.\begin{array}{l} 3 \text{ at } \left(\frac{1}{2}, 1\frac{1}{2}\right) \\ 1 \text{ at } \left(1\frac{1}{2}, \frac{1}{2}\right) \end{array}\right\} = 4 \text{ at } (x, y)$

$3\left(\frac{1}{2}\right) + 1\left(1\frac{1}{2}\right) = 4x$

$\Rightarrow x = \frac{3}{4}$

$3\left(1\frac{1}{2}\right) + 1\left(\frac{1}{2}\right) = 4y$

$\Rightarrow y = 1\frac{3}{4}$



Answer: $\left(\frac{3}{4}, 1\frac{1}{4}\right)$

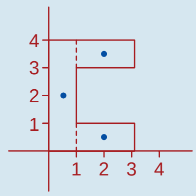
$\tan A = \frac{\frac{3}{4}}{1\frac{3}{4}}$

$= \frac{3}{7}$

$= 0.4286$

$\therefore A = 23° 12'$

(ii)



$\left.\begin{array}{l} 4 \text{ at } \left(\frac{1}{2}, 2\right) \\ 2 \text{ at } \left(2, \frac{1}{2}\right) \\ 2 \text{ at } \left(2, 3\frac{1}{2}\right) \end{array}\right\} = 8 \text{ at } (x, y)$

$4\left(\frac{1}{2}\right) + 2(2) + 2(2) = 8(x)$

$\Rightarrow x = 1\frac{1}{4}$

$4(2) + 2\left(\frac{1}{2}\right) + 2\left(3\frac{1}{2}\right) = 8(y)$

$\Rightarrow y = 2$

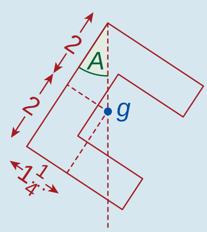
Answer: $\left(1\frac{1}{4}, 2\right)$

$\tan A = \frac{1\frac{1}{4}}{2}$

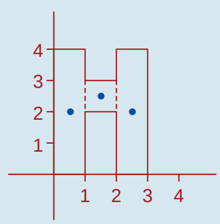
$= \frac{5}{8}$

$= 0.625$

$\therefore A = 32°$



(iii)



$\left.\begin{array}{l} 4 \text{ at } \left(\frac{1}{2}, 2\right) \\ 1 \text{ at } \left(1\frac{1}{2}, 2\frac{1}{2}\right) \\ 4 \text{ at } \left(2\frac{1}{2}, 2\right) \end{array}\right\} = 9 \text{ at } (x, y)$

$4\left(\frac{1}{2}\right) + 1\left(1\frac{1}{2}\right) + 4\left(2\frac{1}{2}\right) = 9(x)$

$\Rightarrow x = \frac{3}{2}$

$$4(2) + 1\left(2\frac{1}{2}\right) + 4(2) = 9(y)$$

$$\Rightarrow y = \frac{37}{18}$$

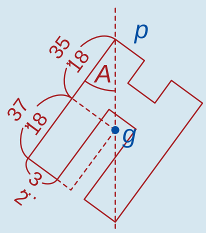
Answer: $\left(\frac{3}{2}, \frac{37}{18}\right)$

$$\tan A = \frac{\frac{3}{2}}{\frac{35}{18}}$$

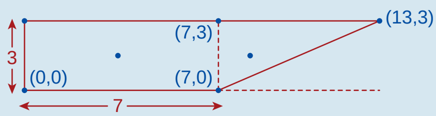
$$= \frac{27}{35}$$

$$= 0.7714$$

$$\therefore A = 37° \, 39'$$



(iv)



Triangle piece:

Centroid is at

$$\left(\frac{7 + 7 + 13}{3}, \frac{3 + 3 + 0}{3}\right) = (9, 2)$$

Area is $\frac{1}{2}(6)(3) = 9$ square units

$$\left.\begin{array}{l} 21 \text{ at } \left(3\frac{1}{2}, 1\frac{1}{2}\right) \\ 9 \text{ at } (9, 2) \end{array}\right\} = 30 \text{ at } (x, y)$$

$$21\left(3\frac{1}{2}\right) + 9(9) = 30(x)$$

$$\Rightarrow x = 5.15$$

$$21\left(1\frac{1}{2}\right) + 9(2) = 30(y)$$

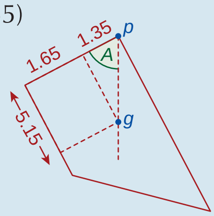
$$\Rightarrow y = 1.65$$

Answer: (5.15, 1.65)

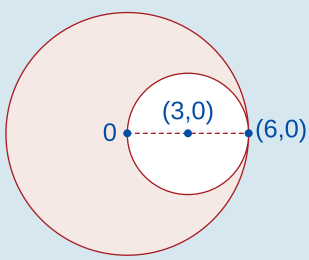
$$\tan A = \frac{5.15}{1.35}$$

$$= 3.8148$$

$$A = 75° \, 19'$$



Q. 2.



Larger Circle: Area $= 36\pi$, Centre $= (0, 0)$

Smaller Circle: Area $= 9\pi$, Centre $= (3, 0)$

Remainder: Area $= 27\pi$,
Centre of gravity is at (x, y)

$$\left.\begin{array}{l} 9\pi \text{ at } (3, 0) \\ 27\pi \text{ at } (x, y) \end{array}\right\} = 36\pi \text{ at } (0, 0)$$

$$\therefore 9\pi(3) + 27\pi(x) = 36\pi(0) \Rightarrow x = -1$$

$$9\pi(0) + 27\pi(y) = 36\pi(0) \Rightarrow y = 0$$

The distance (0, 0) to (−1, 0)

Answer: 1 cm

Q. 3. **Full Square:** Area $= 4 \times 4 = 16$ m^2

Small Square: Area $= 1 \times 1 = 1$ m^2

Remainder: Area $= 16 - 1 = 15$ m^2

$$\left.\begin{array}{l} 1 \text{ at } (3.5, 3.5) \\ 15 \text{ at } (x, y) \end{array}\right\} = 16 \text{ at } (2, 2)$$

$$1(3.5) + 15(x) = 16(2)$$

$$\Rightarrow 3.5 + 15x = 32$$

$$\Rightarrow 15x = 28.5$$

$$\Rightarrow x = 1.9$$

$$1(3.5) + 15(y) = 16(2)$$

$$\Rightarrow y = 1.9$$

$\Rightarrow$ Centre of gravity of the remainder is at (1.9, 1.9)

Q. 4. (i) $\left(\frac{1 + 3 + 5}{3}, \frac{1 + 5 + 0}{3}\right) = (3, 2)$

(ii) $\left(\frac{1 + 9 + 11}{3}, \frac{2 + 6 + 1}{3}\right) = (7, 3)$

(iii) **Square:** Centre of gravity is at (1.5, 1.5)

Area of square $= 3 \times 3 = 9$

Triangle: Centre of gravity is at

$$\left(\frac{3 + 3 + 9}{3}, \frac{0 + 3 + 0}{3}\right) = (5, 1)$$

Area of triangle $= \frac{1}{2}(6)(3) = 9$

$$\left.\begin{array}{l} 9 \text{ at } (1.5, 1.5) \\ 9 \text{ at } (5, 1) \end{array}\right\} = 18 \text{ at } (x, y)$$

$$9(1.5) + 9(5) = 18(x)$$

$$\Rightarrow 18x = 58.5$$

$$\Rightarrow x = 3.25$$

$$9(1.5) + 9(1) = 18(y)$$

$$\Rightarrow 18y = 22.5$$

$$\Rightarrow y = 1.25$$

$\Rightarrow$ Centre of gravity of lamina is at (3.25, 1.25)

(iv) **Rectangle:** Centre of gravity is at (8, 2)

Area of rectangle $= 16 \times 4 = 64$

Triangle: Centre of gravity is at (4, 5)

Area of triangle $= \frac{1}{2}(16)(3) = 24$

$$\left.\begin{array}{l} 64 \text{ at } (8, 2) \\ 24 \text{ at } (8, 5) \end{array}\right\} = 88 \text{ at } (x, y)$$

$64(8) + 24(8) = 88(x)$

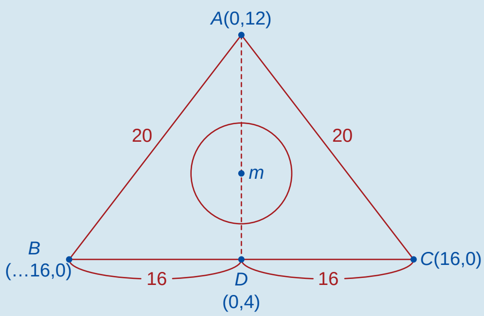
$\Rightarrow x = 8$

$64(2) + 24(5) = 88(y)$

$\Rightarrow y = \frac{31}{11}$

$\Rightarrow$ Centre of gravity of lamina is at $\left(8, \frac{31}{11}\right)$

Q. 5.



$|AB|^2 = |BD|^2 + |AD|^2$

$\Rightarrow 20^2 = 16^2 + |AD|^2$

$\Rightarrow |AD| = 12$

Triangle: Area $= \frac{1}{2}(32)(12)$

$= 192$

Taking D as the origin, the centre of gravity is at the centroid of $A(0, 12)$, $B(-16, 0)$ and C(16, 0) which is at

$$\left(\frac{0 - 16 + 16}{3}, \frac{12 + 0 + 0}{3}\right) = (0, 4)$$

Circle: Area $= \pi r^2$

$= \frac{22}{7} \times \frac{49}{4}$

$= \frac{77}{2}$

$= 38\frac{1}{2}$

Centre of gravity is at $m(0, 6)$

The remainder: Area $= 192 - 38\frac{1}{2}$

$= 153\frac{1}{2}$

Centre of gravity is at (x, y)

$$\left.\begin{array}{l} 153\frac{1}{2} \text{ at } (x, y)\} \\ 38\frac{1}{2} \text{ at } (0, 6) \end{array}\right\} = 192 \text{ at } (0, 4)$$

$153\frac{1}{2}(x) + 38\frac{1}{2}(0) = 192\ (0)$

$\Rightarrow x = 0$

$153\frac{1}{2}(y) + 38\frac{1}{2}(6) = 192(4)$

$\Rightarrow y = \frac{1{,}074}{307}$

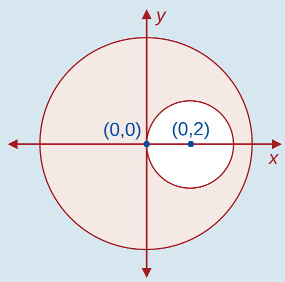
$= 3.5$ cm

Q. 6. Area of full disc $= \pi(5^2) = 25\pi$

Area of missing piece $= \pi(2^2) = 4\pi$

$\Rightarrow$ Area of remainder $= 25\pi - 4\pi$

$= 21\pi$



$$\left.\begin{array}{l} 21\pi \text{ at } (x, y) \\ 4\pi \text{ at } (2, 0) \end{array}\right\} = 25\pi \text{ at } (0, 0)$$

Taking moments around the y-axis:

$21\pi(x) + 4\pi(2) = 25\pi(0)$

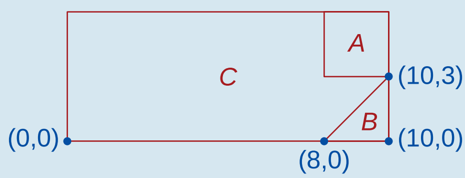
$\Rightarrow 21x + 8 = 0$

$\Rightarrow 21x = -8$

$\Rightarrow x = -\frac{8}{21}$

$\Rightarrow$ Centre of gravity of the remainder is $\frac{8}{21}$ m $= 38$ cm from O.

Q. 7.



A: Area is $3 \times 2 = 6$

Centre of gravity is at $\left(9, 4\frac{1}{2}\right)$

B: Area $= \frac{1}{2}(2)(3) = 3$

Centre of gravity is at

$\left(\frac{(8+10+10)}{3}, \frac{0+0+3}{3}\right) = \left(\frac{28}{3}, 1\right)$

Rectangle: Area $= 10 \times 6 = 60$

Centre of gravity is at $(5, 3)$

Remainder C: Area $= 60 - 6 - 3 = 51.$
Centre of gravity is at (x, y)

$$\left.\begin{array}{l} 6 \text{ at } \left(9, 4\frac{1}{2}\right) \\ 3 \text{ at } \left(\frac{28}{3}, 1\right) \\ 51 \text{ at } (x, y) \end{array}\right\} = 60 \text{ at } (5, 3)$$

$6(9) + 3\left(\frac{28}{3}\right) + 51(x) = 60(5)$

$\Rightarrow x = \frac{218}{51}$

$6\left(4\frac{1}{2}\right) + 3(1) + 51(y) = 60(3)$

$\Rightarrow y = \frac{150}{51}$

Answer: $\left(\frac{218}{51}, \frac{150}{51}\right) = (4.27, 2.94)$

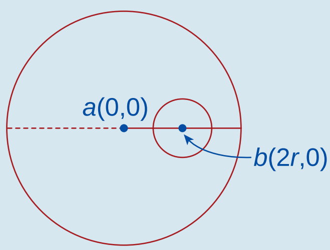
Q. 8. (a) $4(2) + 5(x) + 1(5) + 3(1) = 13(2)$

$\Rightarrow x = 2$

$4(3) + 5(4) + 1(y) + 3(7) = 13(4)$

$\Rightarrow y = -1$

(b)



Larger circle: Area $= \pi R^2$

$= \pi(4r)^2$

$= 16\pi r^2$

Centre of gravity is at $(0, 0)$

Smaller Circle: Area $= \pi r^2$

Centre of gravity is at $b(2r, 0)$

Remainder: Area $= 16\pi r^2 - \pi r^2$

$= 15\pi r^2$

Centre of gravity is at (x, y)

$$\left.\begin{array}{l} 15\pi r^2 \text{ at } (x, y) \\ \pi r^2 \text{ at } (2r, 0) \end{array}\right\} = 16\pi r^2 \text{ at } (0, 0)$$

$15\pi r^2(x) + \pi r^2(2r) = 16\pi r^2\,(0)$

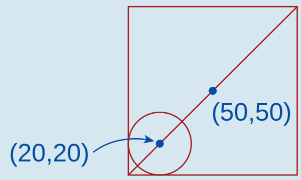
$\Rightarrow x = -\frac{2r}{15}$

$15\pi r^2(y) + \pi r^2(0) = 16\pi r^2\,(0)$

$\Rightarrow y = 0$

Answer: $\left(-\frac{2r}{15}, 0\right)$

Q. 9.



Circle: Area $= \pi r^2$

$= \frac{22}{7} \times \frac{400}{1}$

$= 1{,}257$

Centre of gravity is at $(20, 20)$

Square: Area $= 100 \times 100$

$= 10{,}000$

Centre of gravity is at $(50, 50)$

Remainder: Area $= 10{,}000 - 1{,}257$

$= 8{,}743$

Centre of gravity is at (x, y)

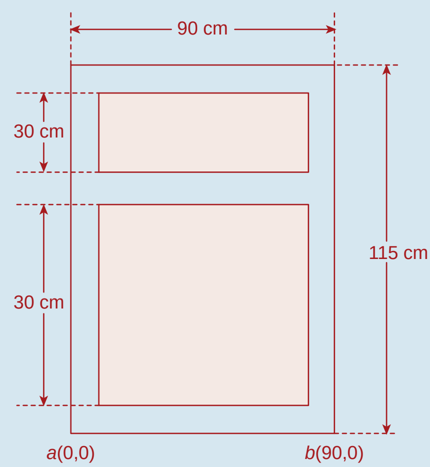
$$\left.\begin{array}{l} 1{,}257 \text{ at } (20, 20) \\ 8{,}743 \text{ at } (x, y) \end{array}\right\} = 10{,}000 \text{ at } (50, 50)$$

$1{,}257(20) + 8{,}743(x) = 10{,}000(50)$

$\Rightarrow x = 54.3$

Answer: 54 mm

Q. 10.



(i) Distance $= \frac{115}{2}$

$= 57.5$ cm

(ii) **Whole rectangle:**

Area $= 90 \times 115$

$= 10{,}350$

Centre of gravity is at (45, 75.5)

Larger Section:

Area $= 80 \times 70$

$= 5{,}600$

Centre of gravity is at (45, 40)

Smaller Section:

Area $= 80 \times 30$

$= 2{,}400$

Centre of gravity is at (45, 95)

Remainder:

Area $= 10{,}350 - 5{,}600 - 2{,}400$

$= 2{,}350$

Centre of gravity is at (x, y)

$$\left.\begin{array}{l} 5{,}600 \text{ at } (45, 40) \\ 2{,}400 \text{ at } (45, 95) \\ 2{,}350 \text{ at } (x, y) \end{array}\right\} = 10{,}350 \text{ at } (45, 57.5)$$

Taking moments about the x-axis:

$5{,}600(40) + 2{,}400(95) + 2{,}350(y) = 10{,}350\,(57.5)$

$\Rightarrow y = 60.9$ cm

Q. 11. (i) Area $\Delta OPQ = \frac{1}{2}$(base)(height) … take [OP] as the base

$\Rightarrow$ Area $\Delta OPQ = \frac{1}{2}(12)(18)$

$= 108$ square units

Area $\Delta OQR = \frac{1}{2}$(base)(height) … take [OR] as the base

$\Rightarrow$ Area $\Delta OQR = \frac{1}{2}(30)(18)$

$= 270$ square units

(ii) Centre of Gravity of ΔOPQ

$= \left(\frac{0 + 0 + 18}{3}, \frac{0 + 12 + 18}{3}\right)$

$= (6, 10)$

Centre of Gravity of ΔOQR

$= \left(\frac{0 + 18 + 30}{3}, \frac{0 + 18 + 0}{3}\right)$

$= (16, 6)$

(iii) $$\left.\begin{array}{l} 108 \text{ at } (6, 10) \\ 270 \text{ at } (16, 6) \end{array}\right\} = 378 \text{ at } (x, y)$$

Taking moments around the *y*-axis:

$(108)(6) + (270)(16) = (378)(x)$

$\Rightarrow 378x = 4{,}968$

$\Rightarrow x = 13.14$

Taking moments around the *x*-axis:

$(108)(10) + (270)(6) = (378)(y)$

$\Rightarrow 378y = 2{,}700$

$\Rightarrow y = 7.14$

$\Rightarrow$ Centre of Gravity of lamina is at (13.14, 7.41)

Q. 12. (i) Area $\Delta OPQ = \frac{1}{2}|x_1y_2 - x_2y_1|$

$= \frac{1}{2}|(18)(6) - (-6)(18)|$

$= 108$

Area $\Delta OQR = \frac{1}{2}$(base)(height) … take [OR] as the base

$\Rightarrow$ Area $\Delta OQR = \frac{1}{2}(36)(18)$

$= 324$ square units

Area ΔOQR : Area $\Delta OQR = 108 : 324$

$= 1 : 3$

(ii) Centre of Gravity of ΔOPQ

$= \left(\frac{0 - 6 + 18}{3}, \frac{0 + 6 + 18}{3}\right)$

$= (4, 8)$

Centre of Gravity of ΔOQR

$= \left(\frac{0 + 18 + 36}{3}, \frac{0 + 18 + 0}{3}\right)$

$= (18, 6)$

(iii) $$\left.\begin{array}{l} 1 \text{ at } (4, 8) \\ 3 \text{ at } (18, 6) \end{array}\right\} = 4 \text{ at } (x, y)$$

Taking moments around the *y*-axis:

$(1)(4) + (3)(18) = (4)(x)$

$\Rightarrow 4x = 58$

$\Rightarrow x = 14.5$

Taking moments around the *x*-axis:

$(1)(8) + (3)(6) = (4)(y)$

$4y = 26$

$\Rightarrow y = 6.5$

$\Rightarrow$ Centre of Gravity of lamina is at (14.5, 6.5)

Exercise 8D

Q. 1. (i) The centre of gravity of the cylinder is at a height $\frac{1}{2}h = \frac{1}{2}(24) = 12$ cm above the table.

(ii) The centre of gravity of the cone is at a height $\frac{1}{4}h = \frac{1}{4}(24) = 6$ cm above the table.

(iii) The centre of gravity of the solid hemisphere is at a height $\frac{3}{8}r = \frac{3}{8}(24) = 9$ cm above the table.

(iv) The centre of gravity of the hemispherical shell is at a height $\frac{1}{2}r = \frac{1}{2}(24) = 12$ cm above the table.

Q. 2. The centre of gravity of the solid hemisphere is at a height $\frac{3}{8}r = \frac{3}{8}(16) = 6$ cm above the table.
The centre of gravity of the hemispherical shell is at a height $\frac{1}{2}r = \frac{1}{2}(16) = 8$ cm above the table.
The difference in the heights of their centres of gravity above the table is $8 - 6 = 2$ cm.

Q. 3. (i) $\frac{1}{4}h = 5$

$\Rightarrow h = 20$ cm

(ii) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(3^2)(20)$

$= 60\pi$ cm^3

Q. 4. (i) $\frac{3}{8}r = 3$

$\Rightarrow r = 8$ cm

(ii) $V = \frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi(8^3)$

$= \frac{1{,}024\pi}{3}$ cm^3

Q. 5. (i) $\frac{1}{2}r = 3$

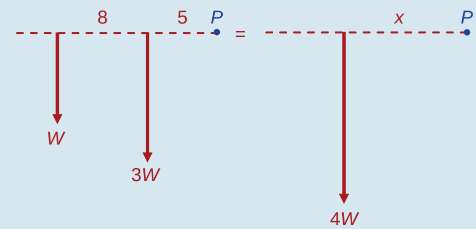
$\Rightarrow r = 6$ cm

(ii) Curved Surface Area $= 2\pi r^2$

$= 2\pi(6^2)$

$= 72\pi$ cm^2

Q. 6. W acts through a point $\frac{3}{8}r = 3$ cm from the base of the cylinder.
$3W$ acts through a point $\frac{1}{2}h = 5$ cm from the base of the hemisphere.
Let P be the point at the right extreme end of the central axis.
Here, then, is the diagram of the forces.
The total weight of the compound body is $4W$ which acts through a point which is x cm from P.



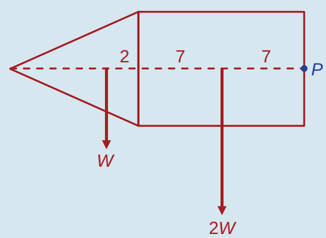
By the Principle of Moments:

$W(13) + 3W(5) = 4W(x)$ … divide by W

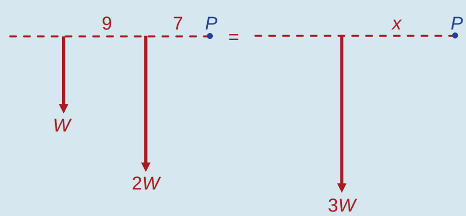
$\Rightarrow 28 = 4x$

$\Rightarrow x = 7$ cm from P

Q. 7.



$2W$ acts through a point $\frac{1}{2}h = \frac{1}{2}(14) = 7$ cm from P.
W acts through a point $\frac{1}{4}(8) = 2$ cm from the base of the cone, i.e. 16 cm from P.
Here, then, is the diagram of the forces.
The total weight of the compound body is $3W$, which acts through a point which is x cm from P.



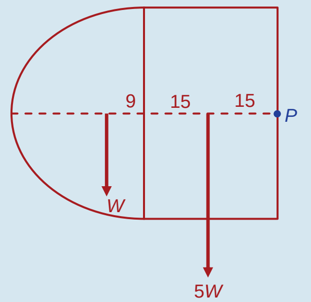
By the Principle of Moments,

$W(16) + 2W(7) = 3W(x)$ … divide by W

$\Rightarrow 30 = 3x$

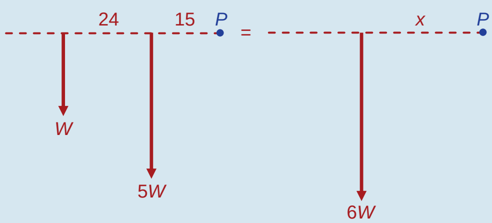
$\Rightarrow x = 10$ cm

Q. 8.



$5W$ acts through a point $\frac{1}{2}h = \frac{1}{2}(30) = 15$ mm from P.
W acts through a point $\frac{3}{8}(24) = 9$ mm from the base of the hemisphere, i.e. 39 mm from P.

Here, then, is the diagram of the forces. The total weight of the compound body is $6W$, which acts through a point which is x mm from P.



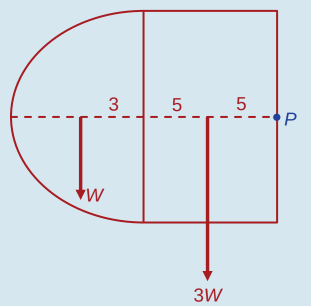
By the Principle of Moments:

$W(39) + 5W(15) = 6W(x)$ … divide by W

$\Rightarrow 114 = 6x$

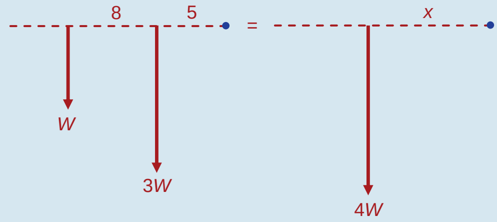
$\Rightarrow x = 19$ mm

Q. 9.



$3W$ acts through a point $\frac{1}{2}h = \frac{1}{2}(10) = 5$ mm from P.
W acts through a point $\frac{1}{2}(6) = 3$ mm from the base of the hemisphere, i.e. 13 mm from P.

Here, then, is the diagram of the forces. The total weight of the compound body is $4W$, which acts through a point which is x mm from P.



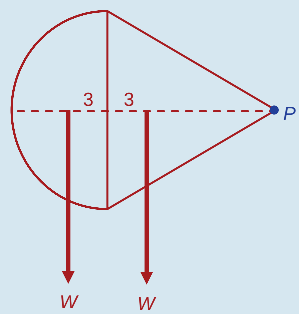
By the Principle of Moments,

$W(13) + 3W(5) = 4W(x)$ … divide by W

$\Rightarrow 28 = 4x$

$\Rightarrow x = 7$ mm

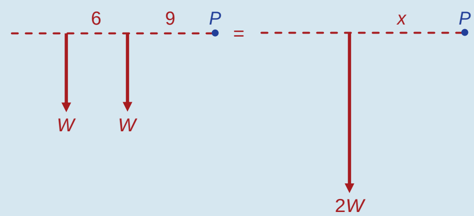
Q. 10.



W acts through a point $\frac{1}{4}(12) = 3$ cm from the base of the hemisphere.

W acts through a point $\frac{3}{8}(8) = 3$ cm from the base of the cone, i.e. 15 cm from P.

Here, then, is the diagram of the forces. The total weight of the compound body is $2W$ which acts through a point which is x cm from P.



By the Principle of Moments,

$W(15) + W(9) = 2W(x)$ … divide by W

$\Rightarrow 24 = 2x$

$\Rightarrow x = 12$ cm

i.e. the centre of the compound body is 12 cm from P. This is at the plane where the bases of the two solids meet.

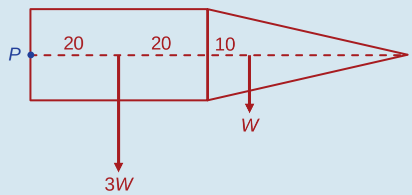
Q. 11. (i) $v_{cylinder} = \pi r^2 h$

$= \pi(12^2)(40)$

$= 5{,}760\pi$ cm^3

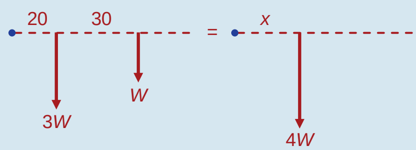
$$v_{cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(12^2)(40)$$
$$= \frac{1}{3}(5{,}760\pi)\ \text{cm}^3$$
$$\Rightarrow v_{cylinder} = 3(v_{cone})$$
$\Rightarrow$ weight of cylinder = 3(weight of cone)

(ii)



Let W be the weight of the cone. Therefore $3W$ is the weight of the cylinder. W acts through a point $\frac{1}{4}h = \frac{1}{4}(40) = 10$ cm from the base of the cylinder. $3W$ acts through a point $\frac{1}{2}h = \frac{1}{2}(40) = 20$ cm from the base of the cone.

Here, then, is the diagram of the forces. The total weight of the compound body is $4W$, which acts through a point which is x cm from P.



By the Principle of Moments:

$3W(20) + W(50) = 4W(x)$

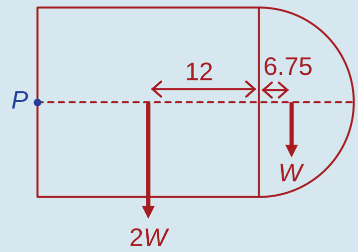
... divide by W

$\Rightarrow 110 = 4x$

$\Rightarrow x = 27.5$ cm

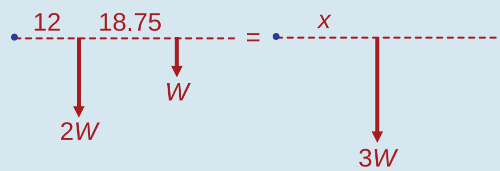
Q. 12. (i) $v_{cylinder} = \pi r^2 h = \pi(18^2)(24)$
$$= 7{,}776\pi\ \text{cm}^3$$
$$v_{hemisphere} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(18^3)$$
$$= 3{,}888\pi\ \text{cm}^3$$
$$\Rightarrow v_{cylinder} = 2(v_{hemisphere})$$
$\Rightarrow$ weight of cylinder : weight of cone = 2 : 1

(ii)



Let W be the weight of the hemisphere. Therefore $2W$ is the weight of the cylinder. W acts through a point $\frac{3}{8}r = \frac{3}{8}(18) = 6{\cdot}75$ cm from the top of the cylinder. $2W$ acts through a point $\frac{1}{2}h = \frac{1}{2}(24) = 12$ cm from the base of the hemisphere.

Here, then, is the diagram of the forces. The total weight of the compound body is $3W$, which acts through a point which is x cm from P.



By the Principle of Moments:

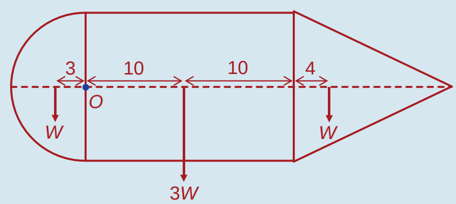
$2W(12) + W(30.75) = 3W(x)$

... divide by W

$\Rightarrow 3x = 54.75$

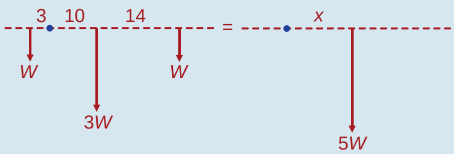
$\Rightarrow x = 18.25$ cm

Q. 13.



Let W be the weight of the hemisphere and of the cone. Therefore $3W$ is the weight of the cylinder. The weight of the hemisphere, W, acts through a point $\frac{3}{8}r = \frac{3}{8}(8) = 3$ cm from the base of the cylinder. The weight of the cylinder, $3W$, acts through a point $\frac{1}{2}h = \frac{1}{2}(20) = 10$ cm from the base of the cylinder. The weight of the cone, W, acts through a point $\frac{1}{4}h = \frac{1}{4}(16) = 4$ cm from the top of the cylinder.

Here, then, is the diagram of the forces. The total weight of the compound body is $5W$ which acts through a point which is x cm from O.



By the Principle of Moments:

$W(-3) + 3W(10) + W(24) = 5W(x)$

… divide by W

$\Rightarrow 5x = 51$

$\Rightarrow x = 10.2$ cm

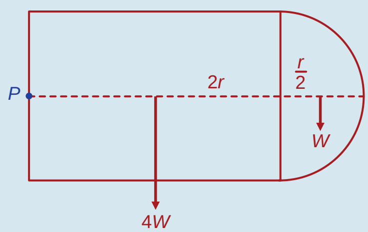
Q. 14. (i) $CSA_{\text{cylinder}} = 2\pi rh = 2\pi(r)(4r) = 8\pi r^2$

$CSA_{\text{hemisphere}} = 2\pi r^2$

$\Rightarrow CSA_{\text{cylinder}} = 4(CSA_{\text{hemisphere}})$

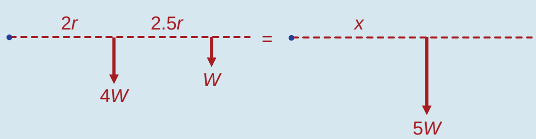
$\Rightarrow W_{\text{cylinder}} = 4(W_{\text{hemisphere}})$

(ii)



Let W be the weight of the hemisphere. Therefore $4W$ is the weight of the cylinder. W acts through a point $\frac{r}{2}$ from the top of the cylinder. $4W$ acts through a point $\frac{1}{2}h = \frac{1}{2}(4r) = 2r$ from the base of the hemisphere.

Here, then, is the diagram of the forces. The total weight of the compound body is $5W$ which acts through a point which is a distance of x from P.



By the Principle of Moments:

$4W(2r) + W(4.5r) = 5W(x)$

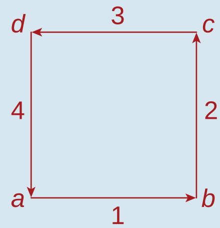
… divide by W

$\Rightarrow 5x = 12.5r$

$\Rightarrow x = 2.5r.$

Exercise 8E

Q. 1.



$\vec{R} = \vec{i} + 2\vec{j} - 3\vec{i} - 4\vec{j}$

$= -2\vec{i} - 2\vec{j}$

$|\vec{R}| = \sqrt{(-2)^2 + (-2)^2}$

$= \sqrt{8}$ N

$\overrightarrow{db} = 3\sqrt{2}\left(\frac{1}{\sqrt{2}}\vec{i}\right) - 3\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\vec{j}$

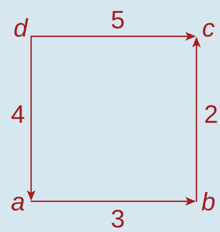
$= 3\vec{i} - 3\vec{j}$

The new resultant $= (-2\vec{i} - 2\vec{j}) + (3\vec{i} - 3\vec{j})$

$= \vec{i} - 5\vec{j}$

$\therefore |\vec{R}| = \sqrt{1^2 + (-5)^2} = \sqrt{26}$ N

Q. 2.



$\vec{R} = 3\vec{i} - 2\vec{i} + 5\vec{i} - 4\vec{j}$

$= 8\vec{i} - 6\vec{j}$

$|\vec{R}| = \sqrt{8^2 + (-6)^2}$

$= \sqrt{100}$

$= 10$ N

Let $x =$ the distance of its line of action from a.

The moment of the sum = the sum of the moments. (Taking moments about a)

$-10(x) = 3(0) - 2(1) - 5(1) + 4(0)$

$\Rightarrow x = \frac{7}{10}$ m

$= 70$ cm

Let it intersect at a distance k from a, therefore a distance $(1 - k)$ from b.

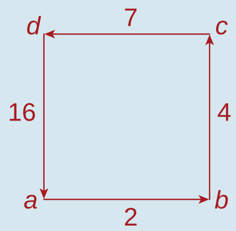
Taking moments about the point of intersection.

$10(0) = 3(0) - 2(1-k) - 5(1) + 4(k)$

$\Rightarrow k = \frac{7}{6}$ m

Answer: $1\frac{1}{6}$ m from a.

Q. 3.



(i) $\vec{R} = 2\vec{i} + 4\vec{j} - 7\vec{i} - 16\vec{j}$

$= -5\vec{i} - 12\vec{j}$

(ii) $\therefore |\vec{R}| = \sqrt{(-5)^2 + (-12)^2}$

$= \sqrt{169}$

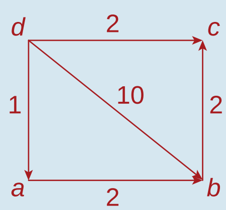
$= 13$ N

(iii) **Taking moments about *a*:**

$13(x) = 2(0) + 4(1) + 7(1) + 16(0)$

$\Rightarrow x = \frac{11}{13}$ m

Q. 4.



(i) $\vec{R} = 2\vec{i} + 2\vec{j} + 2\vec{i} - \vec{j} + (8\vec{i} - 6\vec{j})$

$= 12\vec{i} - 5\vec{j}$

$\therefore |\vec{R}| = \sqrt{12^2 + (-5)^2}$

$= \sqrt{169}$

$= 13$ N

(ii) **Taking moments about *d*:**

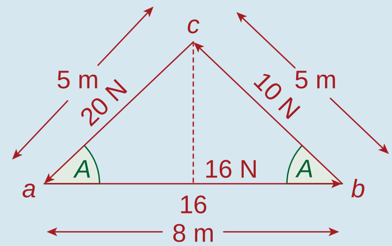
$13(x) = 2(3) + 2(4) + 2(0) + 1(0) + 10(0)$

$\Rightarrow x = \frac{14}{13}$ m

(iii) $13x = 2(3) + 2(4) + 2(0) + 1(0) + 10(0) + 12$

$\Rightarrow x = 2$ m

Q. 5.



$|ad|^2 + |dc|^2 = |ac|^2$

$\Rightarrow 4^2 + |dc|^2 = 5^2$

$\Rightarrow |dc| = 3$ m

$\therefore \tan A = \frac{3}{4}$

$\therefore \sin A = \frac{3}{5}$

$\therefore \cos A = \frac{4}{5}$

$\overrightarrow{ab} = 16\vec{i}$

$\overrightarrow{bc} = -10 \cos A\vec{i} + 10 \sin A\vec{j}$

$= -8\vec{i} + 6\vec{j}$

$\overrightarrow{ca} = -20 \cos A\vec{i} - 20 \sin A\vec{j}$

$= -16\vec{i} - 12\vec{j}$

$\Rightarrow \vec{R} = -8\vec{i} - 6\vec{j}$

$\therefore |\vec{R}| = \sqrt{(-8)^2 + (-6)^2}$

$= \sqrt{100}$

$= 10$ N

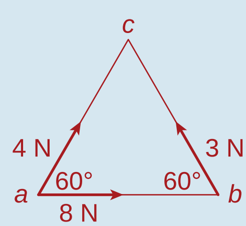
Taking moments about *c*:

Moment of the sum = the sum of the moments

$10(x) = 16(3) + 10(0) + 20(0)$

$\Rightarrow x = 4.8$ m

Q. 6.



(i) $\overrightarrow{ab} = 8\vec{i}$

$\overrightarrow{bc} = -8 \cos 60°\vec{i} + 8 \sin 60°\vec{j}$

$= -8\left(\frac{1}{2}\right)\vec{i} + 8\left(\frac{\sqrt{3}}{2}\right)\vec{j}$

$= -4\vec{i} + 4\sqrt{3}\vec{j}$

$\vec{ca} = 4\cos 60°\vec{i} + 4\sin 60\vec{j}$

$= 2\vec{i} + 2\sqrt{3}\vec{j}$

$\Rightarrow \vec{R} = 6\vec{i} + 6\sqrt{3}\vec{j}$

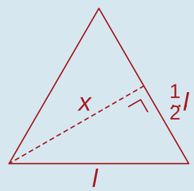
$\therefore |\vec{R}| = \sqrt{6^2 + (6\sqrt{3})^2}$

$= \sqrt{36 + 108}$

$= \sqrt{144}$

$= 12$ N

(ii)



$x^2 + \left(\frac{1}{2}l\right) = l^2$

$\Rightarrow x = \frac{\sqrt{3}}{2}l$

Taking moments about *a*:

Moment of the sum = the sum of the moments

$12(x) = 8(0) + 8\left(\frac{\sqrt{3}}{2}l\right) + 4(0)$

$\Rightarrow 12x = 4\sqrt{3}l$

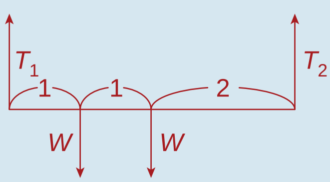
$\Rightarrow x = \frac{\sqrt{3}}{3}l$

$= \frac{1}{\sqrt{3}}l$ m

The moment of the forces about a was $4\sqrt{3}l$ N m, so the moment $M = 4\sqrt{3}l$ N m.

Exercise 8F

Q. 1.



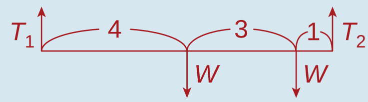
① $T_1 + T_2 = 2W$

② $W(1) + W(2) = T_2(4)$

$\Rightarrow T_2 = \frac{3}{4}W$

$\Rightarrow T_1 = 1\frac{1}{4}W$

Q. 2.



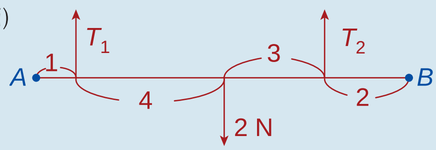
① $T_1 + T_2 = 2W$

② $W(4) + W(7) = T_2(8)$

$\Rightarrow T_2 = \frac{11}{8}W$

$\Rightarrow T_1 = \frac{5}{8}W$

Q. 3. (i)



① $T_1 + T_2 = 2$

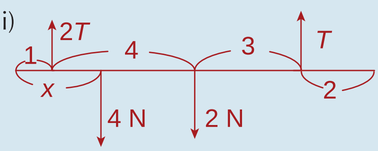
② $2(5) = T_1(1) + T_2(8)$

$\Rightarrow T_1 + T_2 = 10$

Solving these gives $T_1 = \frac{8}{7}$ N,

$T_2 = \frac{6}{7}$ N

(ii)



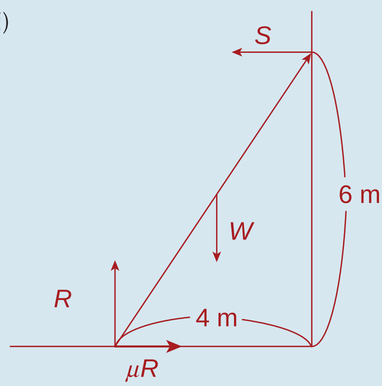
① $T + 2T = 6$

$\Rightarrow T = 2$ N

② $4(x) + 2(5) = 4(1) + 2(8)$

$\Rightarrow x = 2\frac{1}{2}$ cm

Q. 4. (i)



① $R = W$

② $\mu R = S$

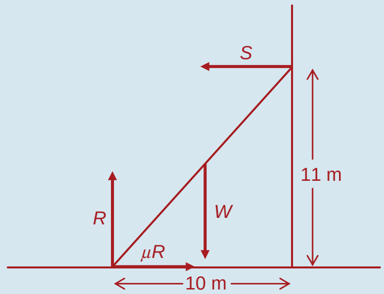
③ $W(2) = S(6)$

$\Rightarrow S = \frac{1}{3}W$

(ii) ② $\mu W = \frac{1}{3}W$

$\Rightarrow \mu = \frac{1}{3}$

Q. 5.



① $R = W$

② $\mu R = S$

③ $W(5) = S(11) \Rightarrow S = \frac{5}{11}W$

② $\mu W = \frac{5}{11}W \Rightarrow \mu = \frac{5}{11}$

Q. 6. (a) (i) Friction

(ii) Moment

(b) ① $R = 245$

② $0.8R = S$

③ $245\ (a \cos \alpha) = S(2a \sin \alpha)$

Equation ②

$\Rightarrow S = 0.8(245) = 196$

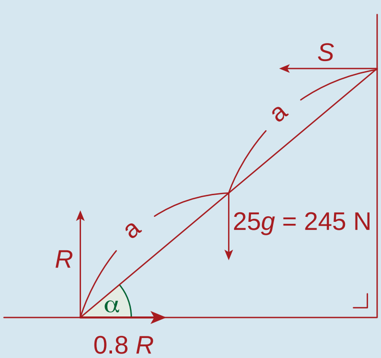
Equation ③

$\Rightarrow 245 \cos \alpha = (196)2 \sin \alpha$

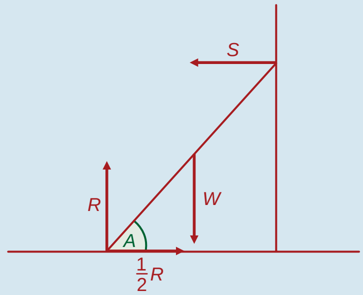
$\Rightarrow 245 = 392 \tan \alpha$

$\Rightarrow \tan \alpha = \frac{245}{392}$

$= \frac{5}{8}$



Q. 7.



Let the ladder have a length of $2l$.

① $R = W$

② $\frac{1}{2}R = S$

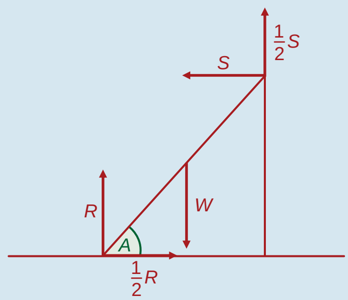
③ $W(l \cos A) = S(2l \sin A)$

$\Rightarrow S = \frac{W \cos A}{2 \sin A} = \frac{W}{2 \tan A}$

② $\frac{1}{2}W = \frac{W}{2 \tan A} \Rightarrow \tan A = 1$

$\Rightarrow A = 45°$.

Q. 8.



Let the ladder have a length of $2l$.

① $R + \frac{1}{2}S = W \Rightarrow R = W - \frac{1}{2}S$

② $\frac{1}{2}R = S$

③ $W(l \cos A) = S(2l \sin A) + \frac{1}{2}S(2l \cos A)$

... divide by $l \cos A$

$\Rightarrow W = 2S \tan A + S$

$\Rightarrow S(2 \tan A + 1) = W$

$\Rightarrow S = \frac{W}{2 \tan A + 1}$

② $\frac{1}{2}\left(W - \frac{1}{2}S\right) = S$

$\Rightarrow \frac{1}{2}W - \frac{1}{4}S = S \Rightarrow 2W - S = 4S$

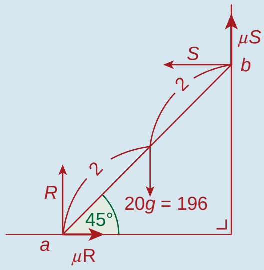
$\Rightarrow 2W = 5S$

$\Rightarrow 2W = \frac{5\ W}{2 \tan A + 1}$

$\Rightarrow 2 \tan A + 1 = \frac{5}{2}$

$\Rightarrow \tan A = \frac{3}{4} \Rightarrow A = 37°$

Q. 9.



① $R + \mu S = 196$

② $\mu R = S$

③ **Taking moments about *a*:**

$196(2 \cos 45°) = S(4 \sin 45°) + \mu S(4 \cos 45°)$

But $\cos 45° = \sin 45° = \frac{1}{\sqrt{2}}$

$\therefore 196(2) = S(4) + \mu S(4)$

$\Rightarrow S + \mu S = 98$

$\Rightarrow S(1 + \mu) = 98$

$\Rightarrow S = \frac{98}{1 + \mu}$

Now $\mu R = S$

$\Rightarrow R = \frac{1}{\mu}S = \frac{1}{\mu}\left(\frac{98}{1 + \mu}\right) = \frac{98}{\mu(1 + \mu)}$

Putting these into equation ①, we get

$\frac{98}{\mu(1 + \mu)} + \mu\left(\frac{98}{1 + \mu}\right) = 196.$

… Multiply by $\mu(1 + \mu)$

$\Rightarrow 98 + 98\mu^2 = 196\mu(1 + \mu).$

… Divide by 98.

$\Rightarrow 1 + \mu^2 = 2\mu(1 + \mu)$

$\Rightarrow 1 + \mu^2 = 2\mu + 2\mu^2$

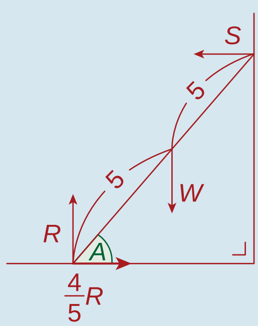
$\Rightarrow \mu^2 + 2\mu - 1 = 0$

$\Rightarrow \mu = \frac{-2 \pm \sqrt{4 + 4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$

$= -1 \pm \sqrt{2}$

Since $\mu > 0$, $\mu = \sqrt{2} - 1$ **Q.E.D.**

Q. 10. (i)



① $R = W$

② $\frac{4}{5}R = S$

③ **Taking moments about the foot of the ladder:**

$W(5 \cos A) = S(10 \sin A)$

$\Rightarrow W \cos A = 2S \sin A$

But $S = \frac{4}{5}R = \frac{4}{5}W.$

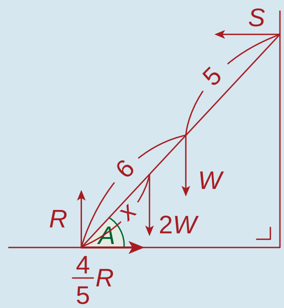
$\therefore W \cos A = 2\left(\frac{4}{5}W\right) \sin A$

$\Rightarrow \cos A = \frac{8}{5} \sin A$

$\Rightarrow 1 = \frac{8}{5} \tan A$

$\Rightarrow \tan A = \frac{5}{8}$

(ii)



Since $\tan A = \frac{3}{4}$, $\cos A = \frac{4}{5}$, $\sin A = \frac{3}{5}$

① $R = 2W + W = 3W$

② $\frac{4}{5}R = S$

$\Rightarrow S = \frac{4}{5}(3W) = \frac{12}{5}W$

③ $2W(x \cos A) + W(5 \cos A)$

$= S(10 \sin A)$

$\Rightarrow 2W\left(\frac{4}{5}x\right) + W(4) = S(6)$

$\Rightarrow \frac{8}{5}xW + 4W = 6S$

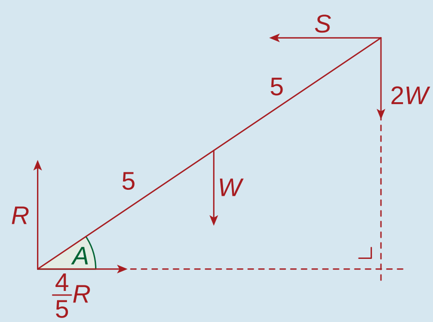
But $S = \frac{12}{5}W$

$\therefore \frac{8}{5}xW + 4W = 6\left(\frac{12}{5}W\right)$

$\Rightarrow 8x + 20 = 72$

$\Rightarrow x = 6.5$ m

(iii)



Assume it is just on the point of slipping when the man reaches the top.

① $R = W + 2W = 3W$

② $\frac{4}{5}R = S$

$\Rightarrow S = \frac{4}{5}(3W) = \frac{12}{5}W$

③ $W(5 \cos A) + 2W(10 \cos A)$

$= S(10 \sin A)$

$\Rightarrow 5W = 2S \tan A$

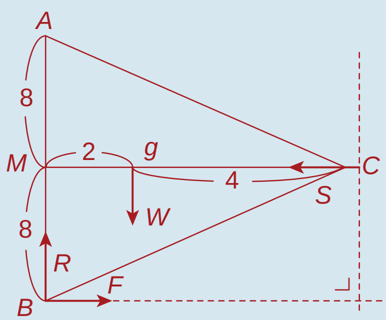
But $S = \frac{12}{5}W$,

$\therefore 5W = \frac{24}{5}W \tan A$

$\Rightarrow \tan A = \frac{25}{24}$

Exercise 8G

Q. 1. (i)



$|MC| = 6$ (from Pythagoras' Theorem)

The centroid, g, is 2 cm from m, 4 cm from C.

Assume it is on the point of slipping. Therefore, $F = \mu R$.

(ii) ① $R = W$

② $\mu R = S$

③ **Taking moments about *b*:**

$W(2) = S(8)$

$\Rightarrow S = \frac{1}{4}W$

Equation ②

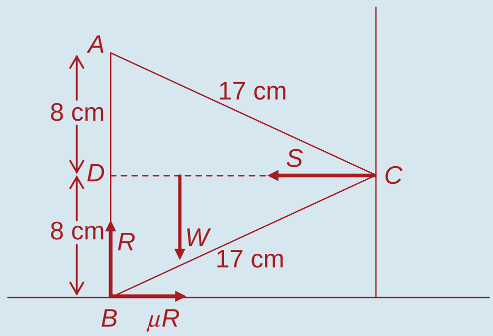
$\Rightarrow \mu R = S$

$\Rightarrow \mu(W) = \frac{1}{4}W$

$\Rightarrow \mu = \frac{1}{4}$

The least value of is $\frac{1}{4}$.

Q. 2.



(i) Using Pythagoras' Theorem

$|CD|^2 + 8^2 = 17^2$

$\Rightarrow |CD| = \sqrt{17^2 - 8^2}$

$= 15$ cm

$[CD]$ is a median of the triangle. The centroid therefore lies $\frac{2}{3}$ of the way along $[CD]$.

$\frac{2}{3}(15) = 10$

$\Rightarrow$ The centre of gravity is 10 cm from C.

(ii) ① $R = W$

② $\mu R = S$

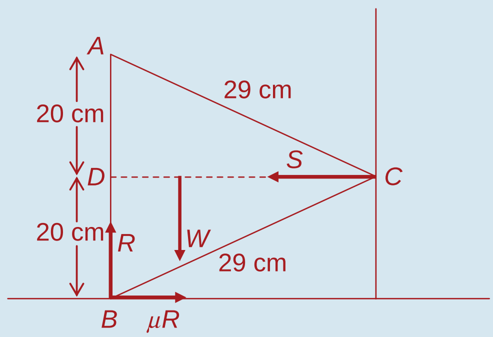
③ $W(5) = S(8)$

$\Rightarrow S = \frac{5}{8}W$

② $\mu W = \frac{5}{8}W$

$\Rightarrow \mu = \frac{5}{8}$

Q. 3.



Using Pythagoras' Theorem

$|CD|^2 + 20^2 = 29^2$

$\Rightarrow |CD| = \sqrt{29^2 - 20^2} = 21$ cm

$[CD]$ is a median of the triangle. The centroid therefore lies $\frac{2}{3}$ of the way along $[CD]$.

$\frac{2}{3}(21) = 14$

$\Rightarrow$ The centre of gravity is 14 cm from C.

① $R = W$

② $\mu R = S$

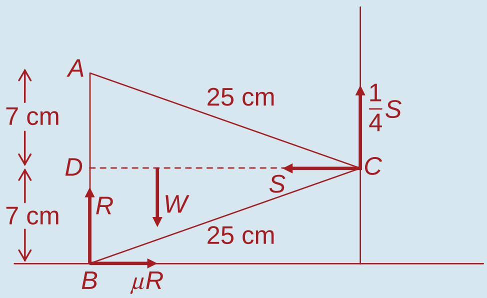
③ $W(7) = S(20)$

$$\Rightarrow S = \frac{7}{20}W$$

② $\mu W = \frac{7}{20}W$

$$\Rightarrow \mu = \frac{7}{20}$$

Q. 4.



(i) Using Pythagoras' Theorem:

$|CD|^2 + 7^2 = 25^2$

$\Rightarrow |CD| = \sqrt{25^2 - 7^2}$

$= 24$ cm

[CD] is the median of the triangle. The centroid therefore lies $\frac{2}{3}$ of the way along [CD].

$\frac{2}{3}(24) = 16$

$\Rightarrow$ The centre of gravity is 16 cm from C.

① $R + \frac{1}{4}S = W$

$\Rightarrow R = W - \frac{1}{4}S$

② $\mu R = S$

③ $W(8) = S(7) + \frac{1}{4}S(24)$

$\Rightarrow 8W = 13S$

$\Rightarrow S = \frac{8}{13}W$

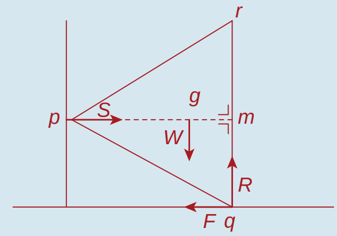
$\Rightarrow R = \frac{11}{13}W$

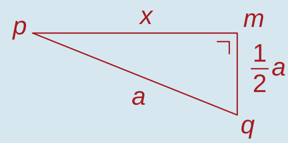
(ii) ② $\mu\left(\frac{11}{13}W\right) = \frac{8}{13}W$

$\Rightarrow \mu = \frac{8}{11}$

Q. 5. (a) See text

(b)





Let $|pm| = x$

$x^2 + \left(\frac{1}{3}a\right)^2 = a^2 \Rightarrow x = \sqrt{\frac{3}{2}}\,a$

$\therefore |pg| = \frac{2}{3}\left(\sqrt{\frac{3}{2}}a\right) = \frac{1}{\sqrt{3}}a$

$|gm| = \frac{1}{3}\left(\frac{\sqrt{3}}{2}a\right) = \frac{1}{2\sqrt{3}}a$

① $R = W$

② $F = S$

③ $S\left(\frac{1}{2}a\right) = W\left(\frac{1}{2\sqrt{3}}a\right)$

$\Rightarrow S = \frac{1}{\sqrt{3}}W$

The reactions are $\frac{1}{\sqrt{3}}W, W$

If it is on the point of slipping, then $F = \mu R$.

Equation 2

$\Rightarrow \mu R = S$

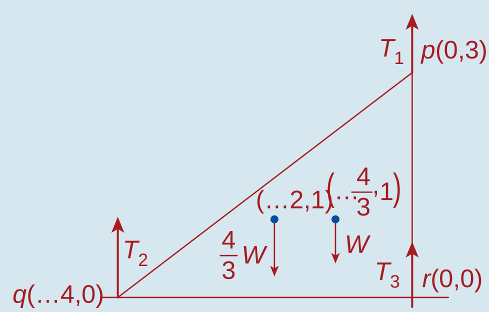
$\Rightarrow \mu W = \frac{1}{\sqrt{3}}W$

$\Rightarrow \mu = \frac{1}{\sqrt{3}}$

$\therefore$ The least value of μ is $\frac{1}{\sqrt{3}}$.

Q. 6. (a) See text

(b)



Centroid is at

$$\left(\frac{0+0+4}{3}, \frac{0+3+0}{3}\right) = \left(-\frac{4}{3}, 1\right)$$

① $T_1 + T_2 + T_3 = \frac{4}{3}W + W$

$\Rightarrow T_1 + T_2 + T_3 = \frac{7}{3}W$

② **Taking moments about the y-axis:**

$T_2(4) + T_1(0) + T_3\ (0) = \frac{4}{3}W(2) + W\left(\frac{4}{3}\right)$

$\Rightarrow 4T_2 = 4W$

$\Rightarrow T_2 = W$

③ **Taking moments about the x-axis:**

$T_1(3) + T_2(0) + T_3(0) = \frac{4}{3}W(1) + W(1)$

$\Rightarrow T_1 = \frac{7}{9}W$

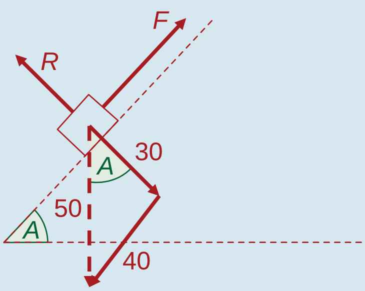
But $T_1 + T_2 + T_3 = \frac{7}{3}W$

$\Rightarrow T_3 = \frac{5}{9}W$

Answer: $\frac{7}{9}W, W, \frac{5}{9}W$

Exercise 8H

Q. 1.



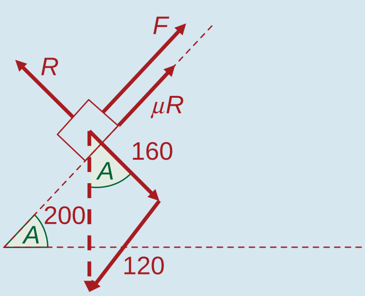
$\tan A = \frac{4}{3}$

$\Rightarrow \cos A = \frac{3}{5}$ and $\sin A = \frac{4}{5}$

Component of weight acting down the plane $= 50 \sin A = 50\left(\frac{4}{5}\right) = 40$ N

$\Rightarrow F = 40$ N

Q. 2.



$\tan A = \frac{3}{4}$

$\Rightarrow \cos A = \frac{4}{5}$ and $\sin A = \frac{3}{5}$

Component of weight acting down the plane $= 200 \sin A = 200\left(\frac{3}{5}\right) = 120$ N

Component of weight acting perpendicular to the plane $= 200 \cos A = 200\left(\frac{4}{5}\right) = 160$ N

$\Rightarrow R = 160$ N

$\Rightarrow$ Friction $= \mu R$

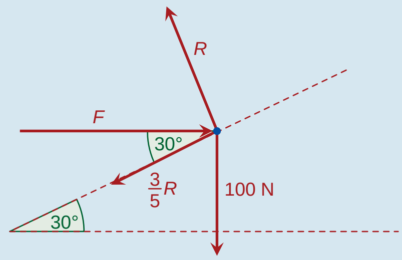
$= \frac{1}{2}(160)$

$= 80$ N

$\Rightarrow F + 80 = 120$

$\Rightarrow F = 40$ N

Q. 3. Here is a diagram of all the forces acting on the particle



Component of weight acting down the plane $= 100 \sin 30° = 100\left(\frac{1}{2}\right) = 50$ N

Component of weight acting perpendicular to the plane $= 100 \cos 30° = 100\left(\frac{\sqrt{3}}{2}\right)$

$= 50\sqrt{3}$ N

Component of F acting along the plane $= F \cos 30° = \frac{\sqrt{3}}{2}F$

Component of F acting perpendicular to the plane $= F \sin 30° = \frac{1}{2}F$

$\Rightarrow R = 50\sqrt{3} + \frac{1}{2}F$

But, $\frac{3}{5}R + 50 = \frac{\sqrt{3}}{2}F$

$\Rightarrow \frac{3}{5}\left[50\sqrt{3} + \frac{1}{2}F\right] + 50 = \frac{\sqrt{3}}{2}F$

$\Rightarrow 30\sqrt{3} + \frac{3}{10}F + 50 = \frac{\sqrt{3}}{2}F$

$\Rightarrow 300\sqrt{3} + 3F + 500 = 5\sqrt{3}F$

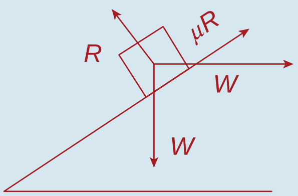
$\Rightarrow F(5\sqrt{3} - 3) = 300\sqrt{3} + 500$

$\Rightarrow F = \frac{100(3\sqrt{3} + 5)}{5\sqrt{3} - 3} \simeq 180$ N

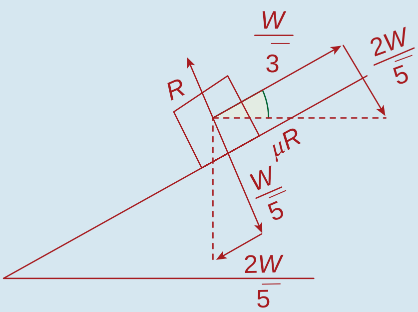
Q. 4. $\tan \alpha = 2$

$\Rightarrow \sin \alpha = \frac{2}{\sqrt{5}} \cos \alpha = \frac{1}{\sqrt{5}}$

Forces



Resolved



① $R = \frac{2W}{\sqrt{5}} + \frac{W}{\sqrt{5}} = \frac{3W}{\sqrt{5}}$

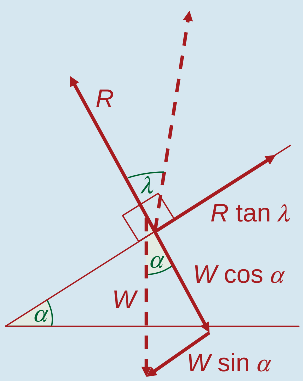
② $\mu R + \frac{W}{\sqrt{5}} = \frac{2W}{\sqrt{5}}$

$\Rightarrow \mu R = \frac{W}{\sqrt{5}}$

Dividing ① by ②

$\Rightarrow \mu = \frac{1}{3}$

Q. 5.



The diagram shows the forces on the particle. Since $\mu = \tan \lambda$, the friction force is $R \tan \lambda$ as shown.

Particle will slip down the plane if

$W \sin \alpha > R \tan \lambda$ ... but $R = W \cos \alpha$

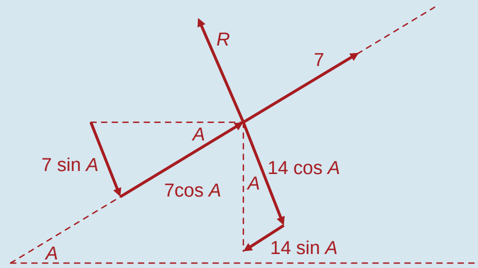
$\Leftrightarrow W \sin \alpha > W \cos \alpha \tan \lambda$

... divide by $W \cos \alpha$

$\Leftrightarrow \tan \alpha > \tan \lambda$

$\Leftrightarrow \alpha > \lambda$

Q. 6. Here is a diagram of the resolved forces acting on the body:



$7 \cos A + 7 = 14 \sin A$

$\Rightarrow \cos A + 1 = 2 \sin A$

$\Rightarrow \cos A + 1 = 2\sqrt{1 - \cos^2 A}$

... square both sides

$\Rightarrow \cos^2 A + 2 \cos A + 1 = 4(1 - \cos^2 A)$

$\Rightarrow \cos^2 A + 2 \cos A + 1 = 4 - 4 \cos^2 A$

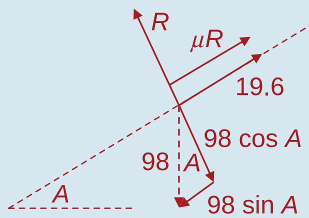
$\Rightarrow 5 \cos^2 A + 2 \cos A - 3 = 0$

$\Rightarrow (5 \cos A - 3)(\cos A + 1) = 0$

$\Rightarrow \cos A = \frac{3}{5}$ **OR** $\cos A = -1$

$\cos A = -1$ is excluded because A is an acute angle.

Q. 7. (i) **Particle about to slip down the plane**



$R = 98 \cos A$

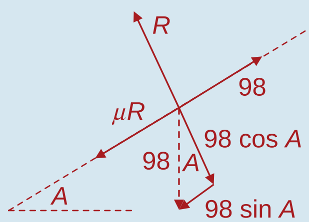
$19.6 + \mu R = 98 \sin A$

$\Rightarrow 19.6 + 98 \mu \cos A = 98 \sin A$

$\Rightarrow 2 + 10 \mu \cos A = 10 \sin A$

$\Rightarrow 10 \sin A - 10 \mu \cos A = 2$ ... ①

Particle about to slip up the plane



$R = 98 \cos A$

$98 = \mu R + 98 \sin A$

$\Rightarrow 98 = 98 \mu \cos A + 98 \sin A$

$\Rightarrow 10 = 10 \mu \cos A + 10 \sin A$

$\Rightarrow 10 \sin A + 10\mu \cos A = 10 \quad \ldots ②$

Adding equations ① and ② we get

$20 \sin A = 12$

$\Rightarrow \sin A = \frac{3}{5}$

$\Rightarrow A = \sin^{-1}\left(\frac{3}{5}\right) = 36.87°$

(ii) From equation ② we get

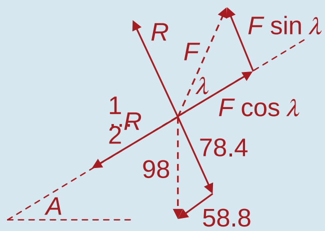
$10\left(\frac{3}{5}\right) + 10\mu\left(\frac{4}{5}\right) = 10$

$\Rightarrow 6 + 8\mu = 10$

$\Rightarrow 8\mu = 4$

$\Rightarrow \mu = \frac{1}{2}$

(iii) The force diagram therefore looks like this:



$R + F \sin \lambda = 78.4$

$\Rightarrow R = 78.4 - F \sin \lambda$

$\frac{1}{2}R + 58.8 = F \cos \lambda$

$\Rightarrow R = 2F \cos \lambda - 117.6$

$\Rightarrow 78.4 - F \sin \lambda = 2F \cos \lambda - 117.6$

$\Rightarrow F(2 \cos \lambda + \sin \lambda) = 196$

$\Rightarrow F = \frac{196}{2 \cos \lambda + \sin \lambda}$

$= 196(2 \cos \lambda + \sin \lambda)^{-1}$

$\Rightarrow \frac{dF}{d\lambda} = -196(2 \cos \lambda + \sin \lambda)^{-2} (-2 \sin \lambda + \cos \lambda)$

$\Rightarrow \frac{dF}{d\lambda} = \frac{196(\cos \lambda - 2 \sin \lambda)}{(2 \cos \lambda + \sin \lambda)^2}$

Putting $\frac{dF}{d\lambda} = 0$ we get

$\cos \lambda - 2 \sin \lambda = 0$

$\Rightarrow 1 - 2 \tan \lambda = 0$

$\Rightarrow \tan \lambda = \frac{1}{2}$

$\Rightarrow \sin \lambda = \frac{1}{\sqrt{5}}$ and $\cos \lambda = \frac{2}{\sqrt{5}}$

$F = \frac{196}{2 \cos \lambda + \sin \lambda}$

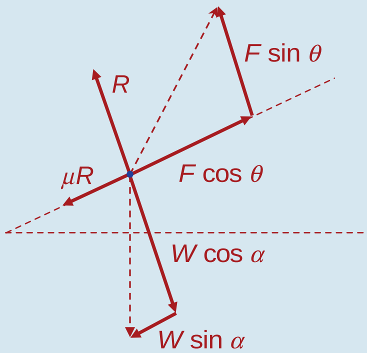
$= \frac{196}{\frac{4}{\sqrt{5}} + \frac{1}{\sqrt{5}}}$

$= \frac{196}{\frac{5}{\sqrt{5}}}$

$= \frac{196}{\sqrt{5}}$ N

Q. 8. $\mu = \tan \lambda$

The diagram shows resolved forces acting on the particle.



$R + F \sin \theta = W \cos \alpha$

$\Rightarrow R = W \cos \alpha - F \sin \theta$

$\mu R + W \sin \alpha = F \cos \theta$

$\Rightarrow R \tan \lambda = F \cos \theta - W \sin \alpha$

$\Rightarrow R = \dfrac{F \cos \theta - W \sin \alpha}{\tan \lambda}$

$$\Rightarrow W \cos \alpha - F \sin \theta = \frac{F \cos \theta - W \sin \alpha}{\tan \lambda}$$

$$\Rightarrow W \cos \alpha - F \sin \theta = (F \cos \theta - W \sin \alpha)\left(\frac{\cos \lambda}{\sin \lambda}\right) \quad \text{... multiply by } \sin \lambda$$

$$\Rightarrow W \cos \alpha \sin \lambda - F \sin \theta \sin \lambda = F \cos \theta \cos \lambda - W \sin \alpha \cos \lambda$$

$$\Rightarrow F(\cos \theta \cos \lambda + \sin \theta \sin \lambda) = W(\cos \alpha \sin \lambda + \sin \alpha \cos \lambda)$$

$$\Rightarrow F \cos(\theta - \lambda) = W \sin(\alpha + \lambda)$$

$$\Rightarrow F = \frac{W \sin(\alpha + \lambda)}{\cos(\theta - \lambda)}$$

(i) Force acting up along the plane $\Rightarrow \theta = 0$

$$\Rightarrow F = \frac{W \sin(\alpha + \lambda)}{\cos(-\lambda)} \quad \dots \cos(-\lambda) = \cos \lambda$$

$$\Rightarrow F = \frac{W \sin(\alpha + \lambda)}{\cos \lambda}$$

(ii) Horizontal force $\Rightarrow \theta = -\alpha$

$$\Rightarrow F = \frac{W \sin(\alpha + \lambda)}{\cos(-\alpha - \lambda)}$$

$$= \frac{W \sin(\alpha + \lambda)}{\cos[-(\alpha + \lambda)]}$$

$$= \frac{W \sin(\alpha + \lambda)}{\cos(\alpha + \lambda)}$$

$$= W \tan(\alpha + \lambda)$$

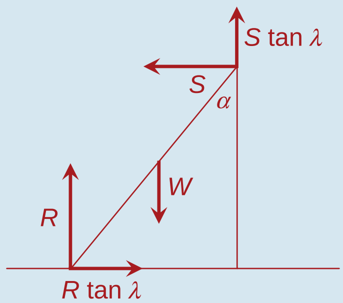
(iii) $F = \dfrac{W \sin(\alpha + \lambda)}{\cos(\theta - \lambda)}$

Minimum force will occur when $\cos(\theta - \lambda)$ is at its maximum value, i.e. $\cos(\theta - \lambda) = 1$

$\Rightarrow F_{MIN} = W \sin(\alpha + \lambda)$.

Q. 9. (i) The normal reaction, R, and the limiting friction, F, acting on a body which is either moving or on the point of moving, can be added to form a resultant. The angle between this resultant and the normal reaction is the **angle of friction**.

(ii)



Let the length of the ladder be $2l$

① $R + S\tan\lambda = W$

$\Rightarrow R = W - S\tan\lambda$

② $R\tan\lambda = S$

③ $Wl\sin\alpha = S(2l\cos\alpha) + S\tan\lambda(2l\sin\alpha)$

$\Rightarrow S(2\cos\alpha + 2\tan\lambda\sin\alpha) = W\sin\alpha$

$$\Rightarrow S = \frac{W\sin\alpha}{2(\cos\alpha + \tan\lambda\sin\alpha)}$$

② $(W - S\tan\lambda)\tan\lambda = S$

$\Rightarrow W\tan\lambda - S\tan^2\lambda = S$

$\Rightarrow S(1 + \tan^2\lambda) = W\tan\lambda$

$$\Rightarrow S = \frac{W\tan\lambda}{1 + \tan^2\lambda}$$

$$\Rightarrow \frac{W\sin\alpha}{2(\cos\alpha + \tan\lambda\sin\alpha)} = \frac{W\tan\lambda}{1 + \tan^2\lambda}$$

$\Rightarrow (1 + \tan^2\lambda)\sin\alpha = 2\tan\lambda(\cos\alpha + \tan\lambda\sin\alpha)$

$\Rightarrow \sin\alpha + \tan^2\lambda\sin\alpha = 2\tan\lambda\cos\alpha + 2\tan^2\lambda\sin\alpha$

$\Rightarrow \tan^2\lambda\sin\alpha + 2\tan\lambda\cos\alpha = \sin\alpha$ ... divide by $\cos\alpha$

$\Rightarrow \tan^2\lambda\tan\alpha + 2\tan\lambda = \tan\alpha$

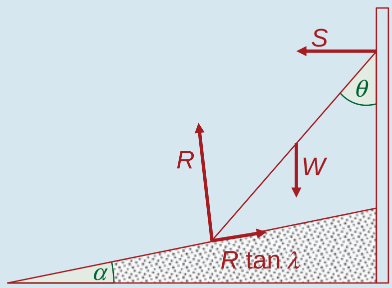
$\Rightarrow \tan\alpha(1 - \tan^2\lambda) = 2\tan\lambda$

$$\Rightarrow \tan\alpha = \frac{2\tan\lambda}{1 - \tan^2\lambda}$$

$\Rightarrow \tan\alpha = \tan 2\lambda$

$\Rightarrow \alpha = 2\lambda$

Q. 10.



Let the length of the ladder be $2l$.

We must firstly resolve R and $R \tan \lambda$ into horizontal and vertical components:

The horizontal component of R is $R \sin \alpha$.

The vertical component of R is $R \cos \alpha$.

The horizontal component of $R \tan \lambda$ is $R \tan \lambda \cos \alpha$.

The vertical component of $R \tan \lambda$ is $R \tan \lambda \sin \alpha$

① $S + R \sin \alpha = R \tan \lambda \cos \alpha$

$$\Rightarrow R = \frac{S}{\tan \lambda \cos \alpha - \sin \alpha}$$

② $W = R \cos \alpha + R \tan \lambda \sin \alpha$

$\Rightarrow W = R(\tan \lambda \sin \alpha + \cos \alpha)$

③ $W(l \sin \theta) = S(2l \cos \theta)$

$\Rightarrow S = \frac{1}{2} W \tan \theta$ … as required

② $W = \left(\frac{S}{\tan \lambda \cos \alpha - \sin \alpha}\right)(\tan \lambda \sin \alpha + \cos \alpha)$

$\Rightarrow W = \frac{1}{2} W \tan \theta \left(\frac{\tan \lambda \sin \alpha + \cos \alpha}{\tan \lambda \cos \alpha - \sin \alpha}\right)$ … divide top and bottom by $\cos \alpha$

$\Rightarrow 1 = \frac{1}{2} \tan \theta \left(\frac{\tan \lambda \tan \alpha + 1}{\tan \lambda - \tan \alpha}\right)$

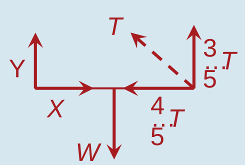
$\Rightarrow \tan \theta(\tan \lambda \tan \alpha + 1) = 2(\tan \lambda - \tan \alpha)$

$\Rightarrow \tan \theta = 2\left(\frac{\tan \lambda - \tan \alpha}{1 + \tan \lambda \tan \alpha}\right)$

$\Rightarrow \tan \theta = 2 \tan(\lambda - \alpha)$ … as required.

Exercise 8I

Q. 1. Here is a diagram of the forces acting on the rod PQ:



① $Y = \frac{3}{5}T = W$ ... **Equation 1**

② $X + \frac{4}{5}T = W$ ... **Equation 2**

③ Take moments about P

$W(2) = \frac{3}{5}T(4)$ ... **Equation 2**

Equation 3: $5W = 6T$

$\Rightarrow T = \frac{5}{6}W$

Equation 1: $Y + \frac{3}{5}\left[\frac{5}{6}W\right] = W$

$\Rightarrow 2Y + W = 2W$

$\Rightarrow 2Y = W$

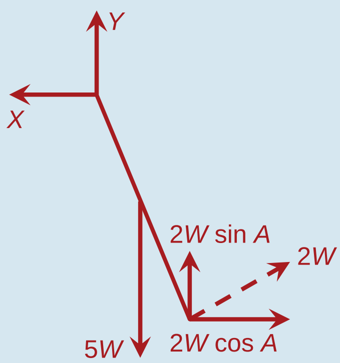
$\Rightarrow Y = \frac{1}{2}W$

Equation 2: $X = \frac{4}{5}\left[\frac{5}{6}W\right]$

$\Rightarrow X = \frac{2}{3}W$

Answer: The horizontal and vertical reactions at the hinge are $\frac{2}{3}W$ and $\frac{1}{2}W$. The tension in the string is $\frac{5}{6}W$.

Q. 2. (i) Here is a diagram of the forces acting on the rod:



① $Y + 2W \sin A = 5W$

② $X = 2W \cos A$

③ Take moments about P

$5W(l \sin A) = 2W(2l)$

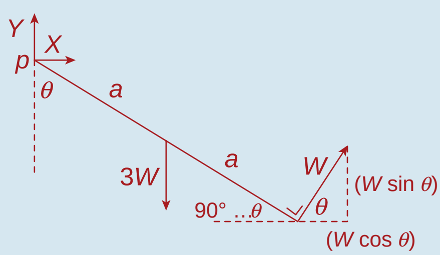
$\Rightarrow \sin A = \frac{4}{5} \qquad \Rightarrow \cos A = \frac{3}{5}$

① $Y + \frac{8}{5}W = 5W \quad \Rightarrow Y = \frac{17}{5}W$

② $X = \frac{6}{5}W$

(ii) $A = \sin^{-1}\frac{4}{5} = 53°$

Q. 3. (i)



(ii) ① $Y + W \sin \theta = 3W$

② $X + W \cos \theta = 0$

③ **Taking moments about *p*, using unresolved forces:**

$3W\,(a \sin \theta) = W(2a)$

$\Rightarrow \sin \theta = \frac{2}{3}$

$= 0.6667$

$\Rightarrow \theta = 41°\ 49'$

(iii) $X = -W \cos \theta$

$= -W\left(\frac{\sqrt{5}}{3}\right)$

$= -\frac{\sqrt{5}}{3}W$

$Y = 3W - W \sin \theta$

$= 3W - W\left(\frac{2}{3}\right)$

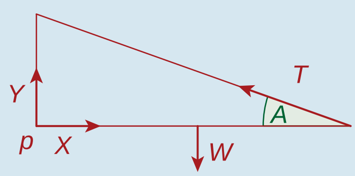
$= \frac{7W}{3}$

Q. 4. Since $\tan A = \frac{5}{12}$

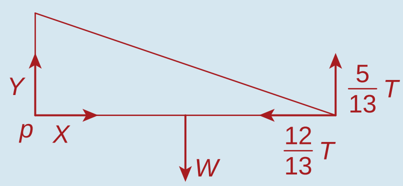
$\cos A = \frac{12}{13}$

$\sin A = \frac{5}{13}$

Forces



Resolved



① $Y + \frac{5}{13}T = W$

② $X = \frac{12}{13}T$

③ Taking moments about p:

$W(6) = \frac{5}{13}T(12)$

$\Rightarrow T = \frac{13}{10}W$

$\therefore X = \frac{12}{13}\left(\frac{13}{10}W\right)$

$= \frac{6}{5}W$

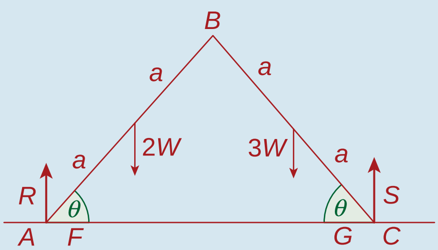
$Y + \frac{5}{13}\left(\frac{13}{10}T\right) = W$

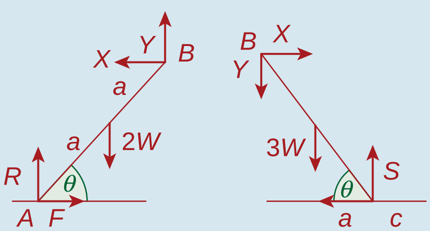
$\Rightarrow Y = \frac{1}{2}W$

Answer: (i) $\frac{6}{5}W, \frac{1}{2}W$

(ii) $\frac{13}{10}W$

Q. 5. (i)





(ii) **From system *ABC***

① $R + S = 2W + 3W$

$\Rightarrow R + S = 5W$

② $F = G$

③ Taking moments about a.

$2W(a \cos \theta) + 3W(3a \cos \theta) = S(4a \cos \theta)$

$\Rightarrow 4S = 11W$

$\Rightarrow S = 2\frac{3}{4}W$

$\therefore R = 2\frac{1}{4}W$, from **Equation 1**.

(iii) Since $R < S, \mu R < \mu S$

$\therefore$ slipping will occur at A first.

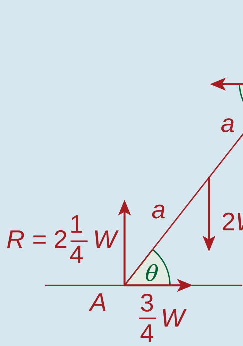
(iv) Let the rod AB be on the point of slipping.

$\therefore F = \mu R$

$= \frac{1}{3}R$

$= \frac{1}{3}\left(2\frac{1}{4}W\right)$

$= \frac{3}{4}W$



① $R + Y = 2W$

② $\frac{3}{4}W = X$

③ Taking moments about B:

$2W(a \cos\theta) + \frac{3}{4}W(2a \sin \theta) = 2\frac{1}{4}(2a \cos \theta)$

$\Rightarrow 2 \cos \theta + \frac{3}{2} \sin\theta = \frac{3}{2} \cos \theta$

$\Rightarrow 3 \sin \theta = 5 \cos\theta$

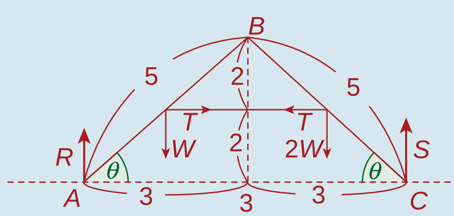
$\Rightarrow 3 \tan \theta = 5$

$\Rightarrow \tan \theta = \frac{5}{3}$

$\Rightarrow \theta = \tan^{-1} \frac{5}{3}$

$= 59°2'$

Q. 6.



Since $\cos \theta = \frac{3}{5}$, $\sin \theta = \frac{4}{5}$ and $\tan \theta = \frac{4}{3}$

The system *ABC*

① $R + S = W + 2W$

$\Rightarrow R + S = 3W$

② $T = T$

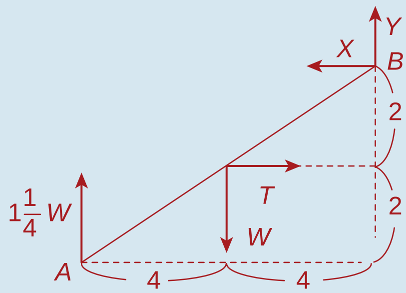
③ Taking moments about A:

$$W\left(1\tfrac{1}{2}\right) + T(2) + 2W\left(4\tfrac{1}{2}\right)$$

$$= T(2) + S(6)$$

$$\Rightarrow S = 1\tfrac{3}{4}W$$

$\therefore R = 1\tfrac{1}{4}W$, from **Equation 1**



The rod *AB*

① $1\tfrac{1}{4}W + Y = W$

$$\Rightarrow Y = -\tfrac{1}{4}W$$

② $T = X$

③ Taking moments about B:

$$T(2) + W\left(1\tfrac{1}{2}\right) = \left(1\tfrac{1}{4}\right)W(3)$$

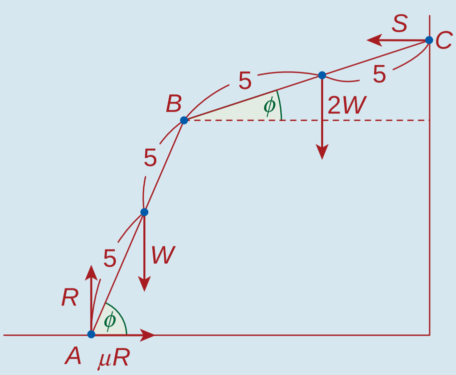
$$\Rightarrow T = 1\tfrac{1}{8}W$$

Answer: $T = 1\tfrac{1}{8}W$,

$R = 1\tfrac{1}{4}W$,

$S = 1\tfrac{3}{4}W$

Q. 7.



Since $\tan\phi = \frac{4}{3}$,

$\sin\phi = \frac{4}{5}$,

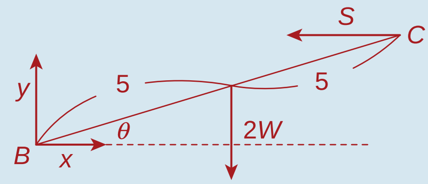
$\cos\phi = \frac{3}{5}$

① $R = W + 2W = 3W$

② $\mu R = S$

③ Taking moments about A:

$$W(5\cos\phi) + 2W(10\cos\phi + 5\cos\theta)$$
$$= S(10\sin\phi + 10\sin\theta)$$
$$\Rightarrow 3W + 12W + 10W\cos\theta$$
$$= 8S + 10S\sin\theta$$
$$\Rightarrow 15W + 10W\cos\theta = 8S + 10S\sin\theta$$



④ $Y = 2W$

⑤ $X = S$

⑥ Taking moments about B:

$$2W(5\cos\theta) = S(10\sin\theta)$$
$$\Rightarrow 10W\cos\theta = 10S\sin\theta$$

This means that Equation ③ reads:

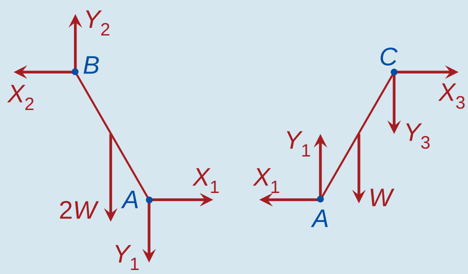
$$15W + 10S\sin\theta = 8S + 10S\sin\theta$$

$$\Rightarrow S = \frac{15}{8}W$$

Equation ② now reads:

$$\mu R = S$$
$$\Rightarrow \mu(3W) = \frac{15}{8}W$$
$$\Rightarrow \mu = \frac{5}{8}$$

Q. 8. (i) Let the length of each rod be $2l$. Here are the forces acting on the rods AB and AC:



Rod *AB*:

Taking moments around B we get:

$$2W(l\cos\beta) + Y_1(2l\cos\beta)$$

$$= X_1(2l\sin\beta) \quad \text{... divide by } 2l\cos\beta$$

$$\Rightarrow W + Y_1 = X_1\tan\beta \quad \text{... \textbf{Equation 1}}$$

Rod AC:

Taking moments around C we get:

$W(l \cos \beta) = X_1(2l \sin \beta) + Y_1 (2l \cos \beta)$

... divide by $l \cos \beta$

$\Rightarrow W = 2X_1 \tan \beta + 2Y_1$

$\Rightarrow W - 2Y_1 = 2X_1 \tan \beta$ ... Equation ②

$2W + 2Y_1 = 2X_1 \tan \beta$ ... Equation ① ($\times$ 2)

Add: $3W = 4X_1, \tan \beta$

$\Rightarrow X_1 = \dfrac{3W}{4 \tan \beta}$ ... substitute into Equation ②

$\Rightarrow W - 2Y_1 = 2\left[\dfrac{3W}{4 \tan \beta}\right] \tan \beta$

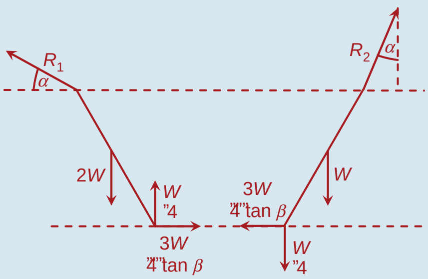
... multiply by 4

$\Rightarrow 4W - 8Y_1 = 6W$

$\Rightarrow 8Y_1 = -2W$

$\Rightarrow Y_1 = -\dfrac{W}{4}$ ... the minus sign indicates that the actual direction of Y_1 is opposite to the direction indicated in the diagram.

(ii) Looking again at the two rods separately, we have the following:



① $R_1 \sin \alpha + \dfrac{W}{4} = 2W$

$\Rightarrow R_1 \sin \alpha = \dfrac{7W}{4}$

② $R_1 \cos \alpha = \dfrac{3W}{4 \tan \beta}$

Dividing Equation ① by Equation ② gives:

$\tan \alpha = \left(\dfrac{7W}{4}\right)\left(\dfrac{4 \tan \beta}{3W}\right)$

$\Rightarrow \tan \alpha = \dfrac{7 \tan \beta}{3}$

③ $R_2 \sin \alpha = \dfrac{3W}{4 \tan \beta}$

④ $R_2 \cos \alpha = W + \dfrac{W}{4}$

$\Rightarrow R_2 \cos \alpha = \dfrac{5W}{4}$

Dividing Equation ③ by Equation ④ gives:

$\tan \alpha = \left(\dfrac{3W}{4 \tan \beta}\right)\left(\dfrac{4}{5W}\right)$

$\Rightarrow \tan \alpha = \dfrac{3}{5 \tan \beta}$

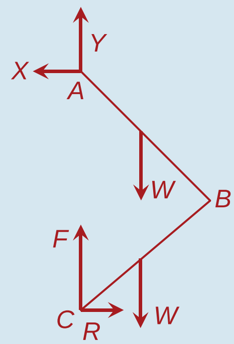
$\Rightarrow \dfrac{7 \tan \beta}{3} = \dfrac{3}{5 \tan \beta}$

$\Rightarrow 35 \tan^2 \beta = 9$

$\Rightarrow \tan^2 \beta = \dfrac{9}{35}$

$\Rightarrow \tan \beta = \dfrac{3}{\sqrt{35}}$

Q. 9. (i) Force diagram for the system including the friction force at C.



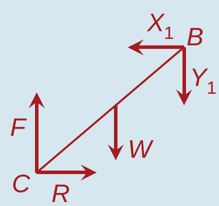
Take moments about A:

$2W\left(\dfrac{l}{2} \sin \phi\right) = R(2l \cos \phi)$

... divide by $l \cos \phi$

$\Rightarrow W \tan \phi = 2R \Rightarrow \quad R = \dfrac{1}{2} W \tan \phi$

Now, we look at the forces on the rod BC in isolation:



Taking moments around B we get:

$F(l \sin \phi) = R(l \cos \phi) + W\left(\dfrac{1}{2} \sin \phi\right)$

... divide by $l \cos \phi$

$\Rightarrow F \tan \phi = R + \dfrac{1}{2} W \tan \phi$

... but $R = \dfrac{1}{2} W \tan \phi$

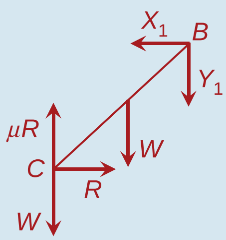
$\Rightarrow F \tan \phi = \dfrac{1}{2} W \tan \phi + \dfrac{1}{2} W \tan \phi$

... divide by $\tan \phi$

$\Rightarrow F = \frac{1}{2}W + \frac{1}{2}W$

$\Rightarrow F = W$

(ii) An additional force is now applied downwards at C. The force diagram for the BC now looks like this:



Because we are now at limiting friction, the friction force is μR.

Taking moments around B we get:

$W(l \sin \phi) + W\left(\frac{1}{2} \sin \phi\right) + R(l \cos \phi)$

$= \mu R(l \sin \phi)$ … divide by $l \cos \phi$

$\Rightarrow W \tan \phi + \frac{1}{2}W \tan \phi + R = \mu R \tan \phi$

… $R = \frac{1}{2}W \tan \phi$

$\Rightarrow W \tan \phi + W \tan \phi = \mu\left(\frac{1}{2}W \tan^2 \phi\right)$

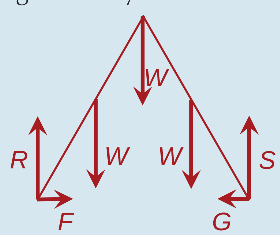
… divide by $W \tan \phi$

$\Rightarrow 2 = \mu\left(\frac{1}{2} \tan \phi\right)$ … multiply by 2

$\Rightarrow \mu \tan \phi = 4$

$\Rightarrow \mu = \frac{4}{\tan \phi}$

Q. 10. **Note:** The person will need to stand at A to maximise the chance of slipping. Because the system will then be symmetrical, slipping will occur at B and C simultaneously. Here are the forces acting on the system:



Taking moments around C we get:

$W\left(\frac{d}{2}\right) + W(d) + W\left(\frac{3d}{2}\right) = R(2d)$

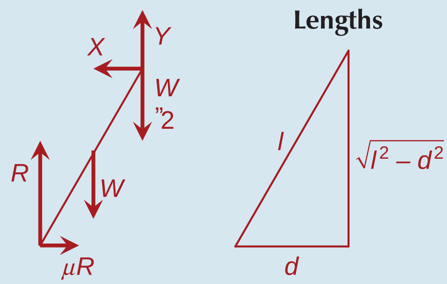
… multiply by $\frac{2}{d}$

$\Rightarrow W + 2W + 3W = 4R$

$\Rightarrow 4R = 6W$

$\Rightarrow R = \frac{3}{2}W$

Now, look at the ladder $[AB]$ in isolation. Here are the forces when it is just on the point of slipping:



Taking moments around A we get:

$R(d) = W\left(\frac{d}{2}\right) + \mu R\left(\sqrt{l^2 - d^2}\right)$

… $R = \frac{3}{2}W$

$\Rightarrow \frac{3}{2}Wd = \frac{1}{2}Wd + \mu\left(\frac{3}{2}W\right)\sqrt{l^2 - d^2}$

… multiply by $\frac{2}{W}$

$\Rightarrow 3d = d + 3\mu\sqrt{l^2 - d^2}$

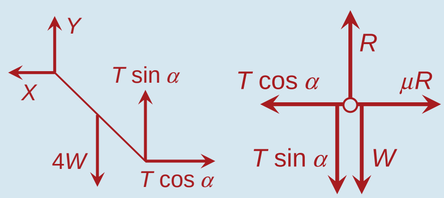
$\Rightarrow 3\mu\sqrt{l^2 - d^2} = 2d$

$\Rightarrow \mu = \frac{2d}{3\sqrt{l^2 - d^2}}$

$\Rightarrow$ In order to avoid slipping

$\mu \geq 2\frac{d}{3\sqrt{l^2 - d^2}}$

Q. 11. (i) **Rod [AB]** **Ring C**



Taking moments about A we get:

$4W(l \cos\alpha) = T \cos \alpha(2l \sin \alpha) + T \sin \alpha(2l \cos \alpha)$

… divide by $2l \cos \alpha$

$\Rightarrow 2W = T \sin \alpha + T \sin \alpha$

$\Rightarrow 2W = 2T \sin \alpha$

$\Rightarrow W = T \sin \alpha$

From the diagram of forces on the ring C we can see that

$\mu R = T \cos \alpha$ ... Equation (1)

and $R = T \sin \alpha + W$

... but $W = T \sin \alpha$

$\Rightarrow R = 2T \sin \alpha$ ... Equation (2)

Dividing Equation (1) by Equation (2) we get:

$\mu = \dfrac{1}{2 \tan \alpha}$

$\Rightarrow 2\mu \tan \alpha = 1$

$\Rightarrow \tan \alpha = \dfrac{1}{2\mu}$ ... (i)

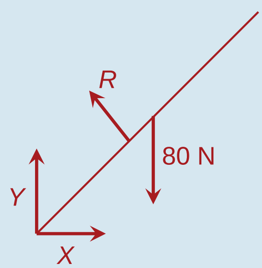
(ii) $W = T \sin \alpha$

$\Rightarrow T = W/\sin \alpha$ ... $\tan \alpha = \dfrac{1}{2\mu}$

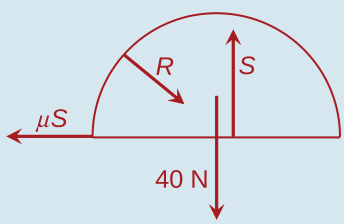
$\Rightarrow \sin \alpha = \dfrac{1}{\sqrt{1 + 4\mu^2}}$

$\Rightarrow T = W\sqrt{1 + 4\mu^2}$

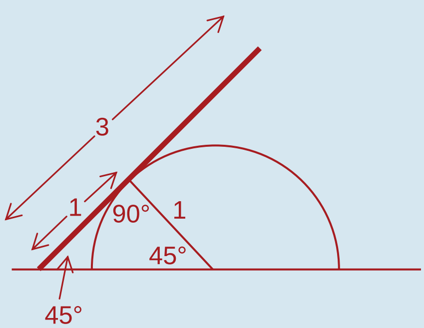
Q. 12. (i) **Rod**



Hemisphere



Lengths



(ii) Taking moments around the hinge we get:

$80\left(\dfrac{1.5}{\sqrt{2}}\right) = R(1)$

$R = \dfrac{120}{\sqrt{2}} = 60\sqrt{2}$ N

(iii) $S = \dfrac{R}{\sqrt{2}} + 40$

$\mu S = \dfrac{R}{\sqrt{2}}$

$\Rightarrow \mu\left(\dfrac{R}{\sqrt{2}} + 40\right) = \dfrac{R}{\sqrt{2}}$

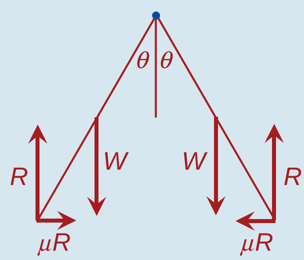
... but $R = 60\sqrt{2}$

$\Rightarrow \mu(60 + 40) = 60$

$\Rightarrow 100\mu = 60$

$\Rightarrow \mu = 0.6$

Q. 13. Assume ladders are on the point of slipping. We will then be at the smallest value of μ that will prevent slipping from occurring. Here is the force diagram for the system:

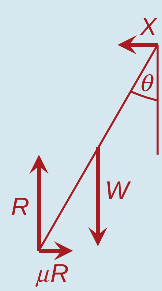


Note: Because both ladders have the same weight, both reaction forces are the same and slipping occurs simultaneously at A and C.

$2R = 2W$

$\Rightarrow R = W$

Now, we examine the forces on the ladder $[AB]$ in isolation:



X is the supporting force from the other ladder. There is no Y component as the vectors in the Y direction already sum to zero.

Taking moments around B we get:

$R(l \sin\theta) = \mu R(l \cos\theta) + W\left(\frac{l}{2}\sin\theta\right)$

… multiply by $\frac{2}{l\cos\theta}$ and let $W = R$

$\Rightarrow 2R\tan\theta = 2\mu R + R\tan\theta$

… divide by R

$\Rightarrow 2\tan\theta = 2\mu + \tan\theta$

$\Rightarrow 2\mu = \tan\theta$

$\Rightarrow \mu = \frac{1}{2}\tan\theta$

This is the minimum value of θ that will prevent slipping from occurring.

$\Rightarrow \mu \geq \frac{1}{2}\tan\theta$

Q. 14. (i) Let x equal the distance from P to where the block touches the rod.

$\sin\alpha = \frac{0.8y}{x}$ … $\tan\alpha = \frac{4}{3}$

$\Rightarrow \sin\alpha = \frac{4}{5}$

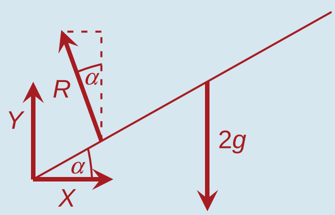
$\Rightarrow \frac{4}{5} = \frac{0.8y}{x}$

… multiply both sides by $5x$

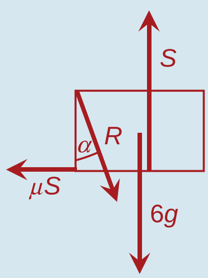
$\Rightarrow 4x = 4y$

$\Rightarrow x = y$

(ii) **Rod**



Block



(iii) Firstly, examine the diagram of the rod. Taking moments around the hinge we get:

$2g(3\cos\alpha) = R(1)$ … $\cos\alpha = \frac{3}{5}$

$\Rightarrow R = \frac{18}{5}g$

Now, examine the diagram of the block:

① $S = R\cos\alpha + 6g$

$\Rightarrow S = \left(\frac{18}{5}g\right)\left(\frac{3}{5}\right) + 6g$

$\Rightarrow S = \frac{204}{25}g$

② $\mu S = R\sin\alpha$

$\Rightarrow \mu\left(\frac{204}{25}g\right) = \left(\frac{18}{5}g\right)\left(\frac{4}{5}\right)$

… multiply by $\frac{25}{g}$

$\Rightarrow 204\mu = 72$

$\Rightarrow \mu = \frac{72}{204} = \frac{6}{17}$

(iv) Looking at the diagram of the rod:

$X = R\sin\alpha$

$= \left(\frac{18}{5}g\right)\left(\frac{4}{5}\right)$

$= 28.224$

$Y + R\cos\alpha = 2g$

$\Rightarrow Y = 2g - R\cos\alpha$

$= 2g - \left(\frac{18}{5}g\right)\left(\frac{3}{5}\right)$

$= -1.568$

… i.e. 1.568 N downwards

$\Rightarrow$ Reaction at hinge

$= 28.224\vec{i} - 1.568\vec{j}$ N

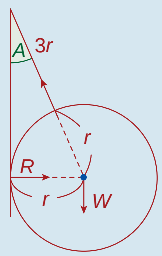
Magnitude

$= \sqrt{(28.224)^2 + (1.568)^2}$

$= 28$ N

Exercise 8J

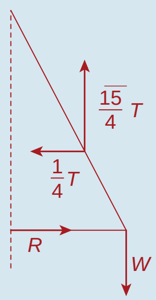
Q. 1. **Forces:**



$$\sin A = \frac{r}{3r + r} = \frac{1}{4}$$

$$\therefore \cos A = \frac{\sqrt{15}}{4}$$

Resolved Forces:



(1) $\frac{\sqrt{15}}{4}T = W$

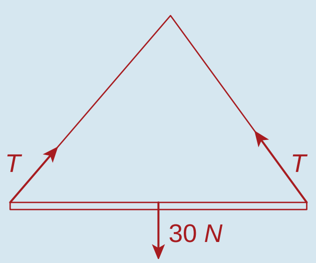
$\Rightarrow T = \frac{4W}{\sqrt{15}}$

(2) $R = \frac{1}{4}T$

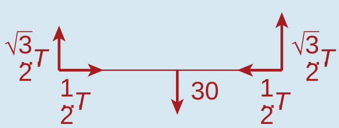
$= \frac{1}{4}\left(\frac{4W}{\sqrt{15}}\right)$

$= \frac{W}{\sqrt{15}}$

Q. 2. **Forces:**



Resolved Forces:



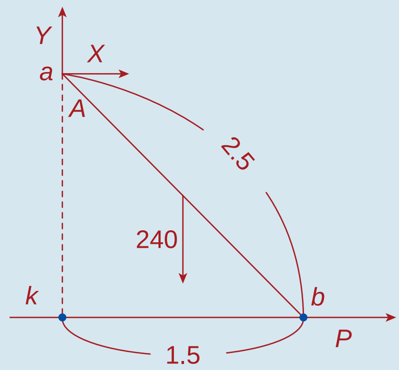
(1) $\frac{\sqrt{3}}{2}T + \frac{\sqrt{3}}{2}T = 30$

$\Rightarrow T = \frac{30}{\sqrt{3}}$

$= 10\sqrt{3}\ N$

Q. 3. (i) "....... are concurrent"

(ii)



$|ak|^2 + |kb|^2 = |ab|^2$

$\Rightarrow |ak|^2 + 2.25 = 6.25$

$\Rightarrow |ak| = 2$

(1) $Y = 240$

(2) $P + X = 0 \Rightarrow X = -P$

(3) **Taking moments about *a*:**

$240(0.75) = P(2)$

$\Rightarrow P = 90\ N$

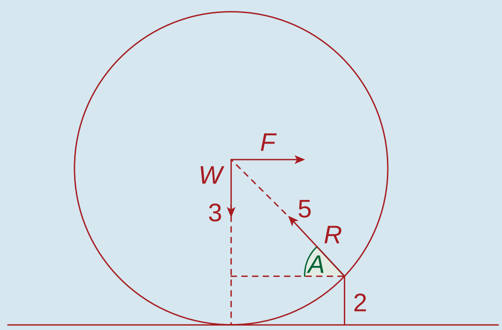
$\vec{X} = -90\vec{i},\ \vec{Y} = 240\vec{j}$

Resultant $= -90\vec{i} + 240\vec{j} = |\vec{R}|$

$= \sqrt{(-90)^2 + (240)^2}$

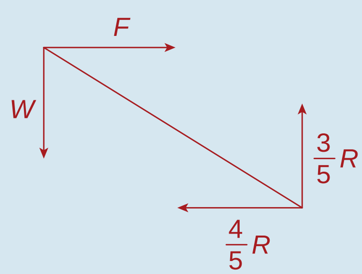
$= 256\ N$

Q. 4. (i) **Forces:**



Since $\sin A = \frac{3}{5}$, $\cos A = \frac{4}{5}$

Resolved Forces:

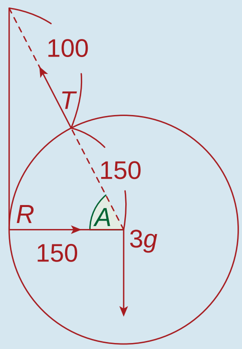


(ii) In accordance with Theorem 8.7

(iii) (1) $\frac{3}{5}R = W \Rightarrow R = \frac{5}{3}W$

(2) $F = \frac{4}{5}R = \frac{4}{5}\left(\frac{5}{3}W\right) = \frac{4}{3}W$

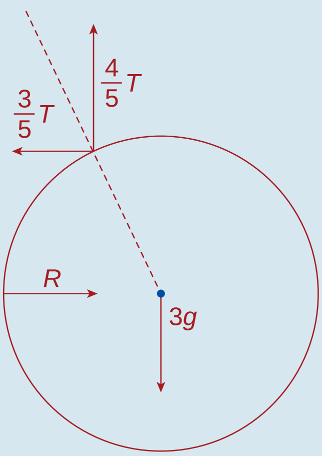
Q. 5. (a) **Forces:**



$\cos A = \frac{150}{250} = \frac{3}{5}$

$\therefore \sin A = \frac{4}{5}$

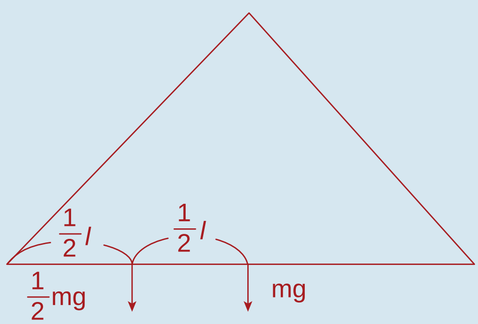
Resolved Forces:



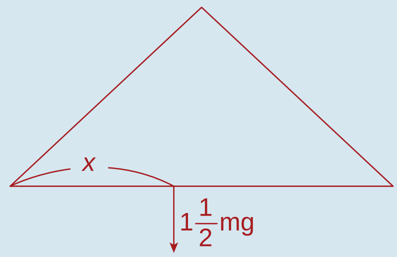
(1) $\frac{4}{5}T = 3g \Rightarrow T = \frac{15}{4}g$

Q. 6. **Step 1.** To find centre of gravity:

Forces:



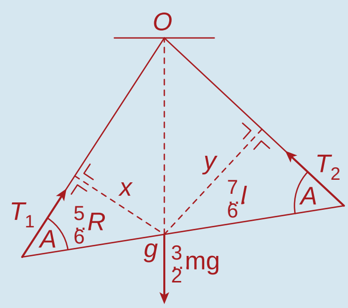
Resultant:



The sum of the moment = the moment of the sum

$\frac{1}{2}\text{mg}\left(\frac{1}{2}l\right) + \text{mg}(l) = 1\frac{1}{2}\text{mg}(x)$

$\Rightarrow x = \frac{5}{6}l$



The resultant weight, $\frac{3}{2}$mg acts through g, which must be below in accordance with Theorem 8.7

Taking moments about g:

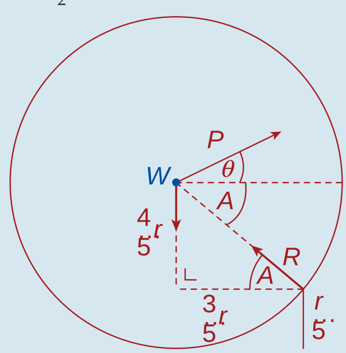
$T_1(x) = T_2(y)$

But $x = \frac{5}{6}l \sin A$, $y = \frac{7}{6}l \sin A$

$\therefore T_1\left(\frac{5}{6}l \sin A\right) = T_2\left(\frac{7}{6}l \sin A\right)$

$\Rightarrow \frac{T_1}{T_2} = \frac{7}{5}$

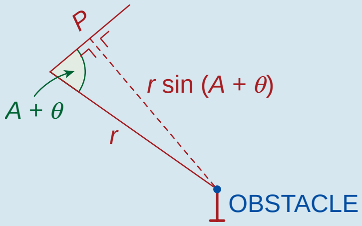
Q. 7.



$\sin A = \frac{4}{5}$

$\therefore \cos A = \frac{3}{5}$

Taking moments about the top of the obstacle:



$W\left(\frac{3}{5}r\right) = P(r\sin(A+\theta))$

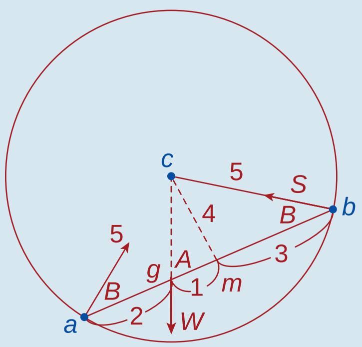
$\Rightarrow P = \dfrac{3W}{5\sin(A+\theta)}$

(i) In this case $\theta = 0$

$\therefore P = \dfrac{3W}{5\sin A} = \dfrac{3W}{4}$

(ii) In this case we want a minimum value for $\dfrac{3W}{5\sin(A+\theta)}$. This value is attained when $\sin(A+\theta) = 1$, and is $\dfrac{3W}{5}$

Q. 8.



Let m be the midpoint $[ab]$.

$|cm| = 4$ (by Pythagoras), $|gm| = 1$ and $|gc| = \sqrt{17}$

$\sin A = \dfrac{4}{\sqrt{17}}$ and $\sin B = \dfrac{4}{5}$

Taking moments about *a*:

$W(2\sin A) = S(6\sin B)$

$\Rightarrow W\left(\frac{8}{\sqrt{17}}\right) = S\left(\frac{24}{5}\right)$

$\Rightarrow S = \dfrac{5W}{3\sqrt{17}}$

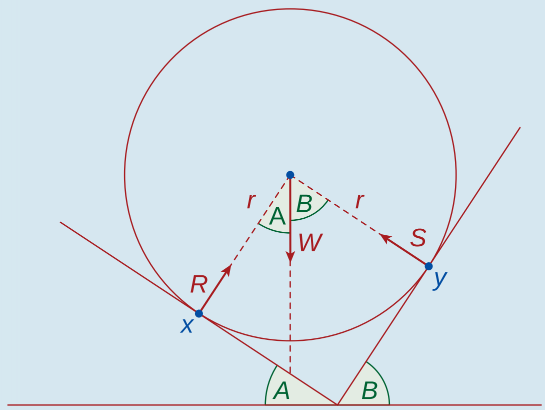
Taking moments about *b*:

$W(4\sin A) = S(6\sin B)$

$\Rightarrow W\left(\frac{16}{\sqrt{17}}\right) = S\left(\frac{24}{5}\right)$

$\Rightarrow R = \dfrac{10W}{3\sqrt{17}}$

Q. 9.



Since $\tan A = \frac{1}{2}$, $\sin A = \frac{1}{\sqrt{5}}$ and $\cos A = \frac{2}{\sqrt{5}}$

Since $\tan B = \frac{3}{4}$, $\sin B = \frac{3}{5}$ and $\cos B = \frac{4}{5}$

Also, $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$$= \left(\frac{1}{\sqrt{5}}\right)\left(\frac{4}{5}\right) + \left(\frac{2}{\sqrt{5}}\right)\left(\frac{3}{5}\right)$$

$$= \frac{10}{5\sqrt{5}}$$

$$= \frac{2}{\sqrt{5}}$$

Taking moments about *x*:

$W(r\sin A) = S(r\sin(A+B))$

$\Rightarrow W\left(\frac{1}{\sqrt{5}}r\right) = S\left(\frac{2}{\sqrt{5}}r\right)$

$\Rightarrow S = \frac{1}{2}W$ **Q.E.D.**